TRISECTING AN ANGLE, USING ONLY AN UNMARKED STRAIGHT EDGE AND A COMPASS. “IMPOSSIBLE”, SAY THE EXPERTS - “SIMPLE “, SAYS THIS TRISECTOR

(Mystery and Solution)

ABSTRACT: This thesis presents solutions to four drafting puzzles that date back to Ancient Greece and are universally considered to be insolvable. These are: Trisecting an Angle: Constructing a Nonagon: Squaring the Circle and Doubling the Cube. These seemingly simple tasks are problems because the draftsman can use only a compass and an unmarked straight edge as his drawing tools. Remarkably, the graphical solutions illustrated in this paper show that each puzzle has a ***single*** *line solution which properly located on its basic puzzle diagram, proves to be the key to/or the direct solution to that geometric problem. For each graphical solution drawings are shown and where necessary, mathematical equations are presented to substantiate the accuracy of the solutions. Finally, the writer concludes that while the solutions submitted are an achievement, the study and methodological approaches taken to solve these ancient problems are instructive and have as much merit as the solutions.*

Foreword: With but a single line drawn, there began a journey toward the exact solutions to four 2,500-year-old Greek graphical puzzles. It started during a search through Wikipedia where I came across a site that listed six unsolved problems. Trisecting an Angle-one of the Greek puzzles - was listed. I then decided to work on this problem. However, Wikipedia, whose web site presents and lists several papers discussing this topic, concludes that this trisecting problem has no solution! More significantly is the book "THE TRISECTORS" by Underwood Dudley, in which he writes of many historic notables who have tried to solve this problem only to have woefully admitted to having failed. In his heavily researched book Professor Dudley comments on and illustrates a number of more recent attempts at trisecting an angle. Moreover, his index alone, which lists impressive names, societies and bibliographies, would cause many to accede to his admonishing conclusion: **"you can't Trisect angles, don't try**". But as it is often the case, curiosity has no bounds and like countless others throughout the centuries, I had to try the impossible. So I looked for a solution and- improbably enough- found it!

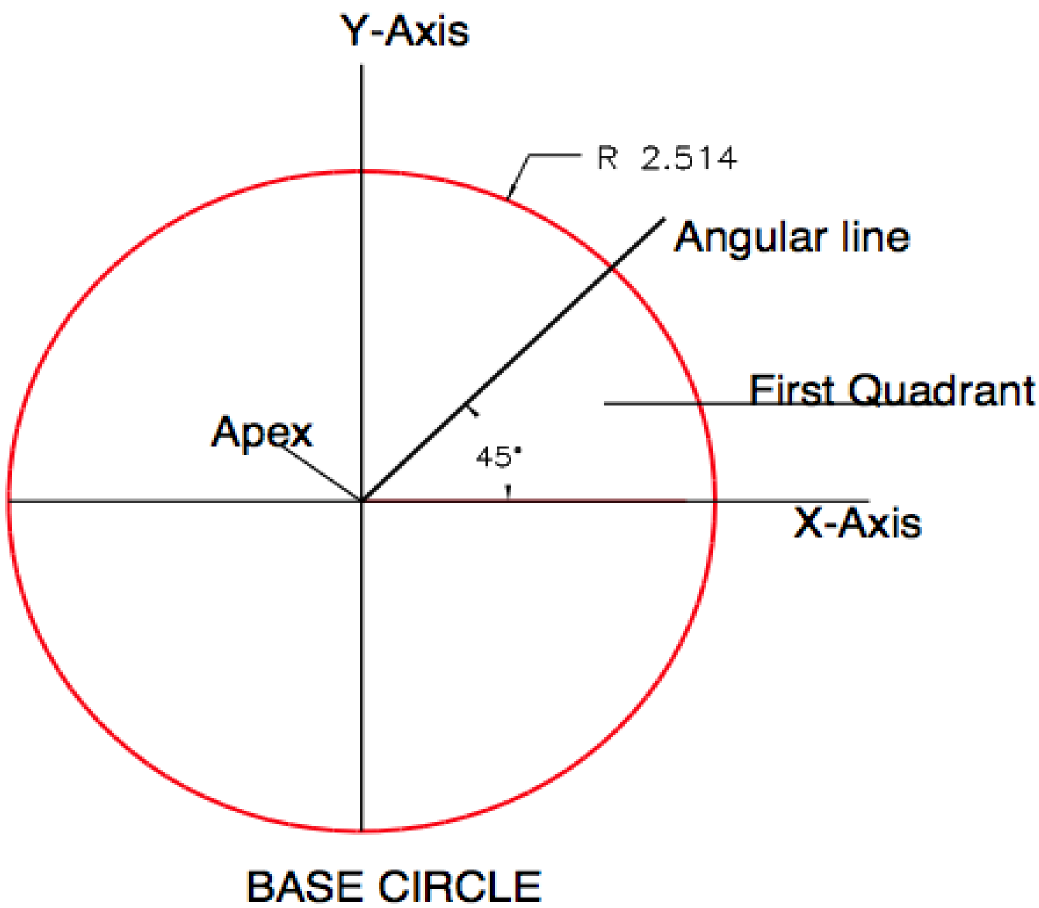
And while this paper presents an exact solution to this 2,500-year-old problem - for those not familiar with this subject matter, the Trisectors (those who try to trisect an angle), the naysayers and the mathematicians, this paper contains illustrations, historical references, equations and graphs which will show why the solution presented is the only solution possible!

Introduction: In my long experience as an analytical engineer, at some point I concluded that there are no **simple problems**, only **simple solutions** and that one should fully understand the problem before attempting to find that solution. So, in thinking about this 2500-year-old problem, I wondered what information or approach was there about this seemingly simple puzzle that was overlooked or missing. After further thought, it seemed that the best approach to understanding this drafting puzzle was to start with a trisected angle model drawing. And to reduce the problem further, worked only in that truly magical first quadrant of a circle. After drawing a trisected angle in that quadrant using the required tools, with but a single horizontal line I was sure I had found my “Simple” solution to this ancient puzzle and with it, the basic steps to a mathematical solution as well.

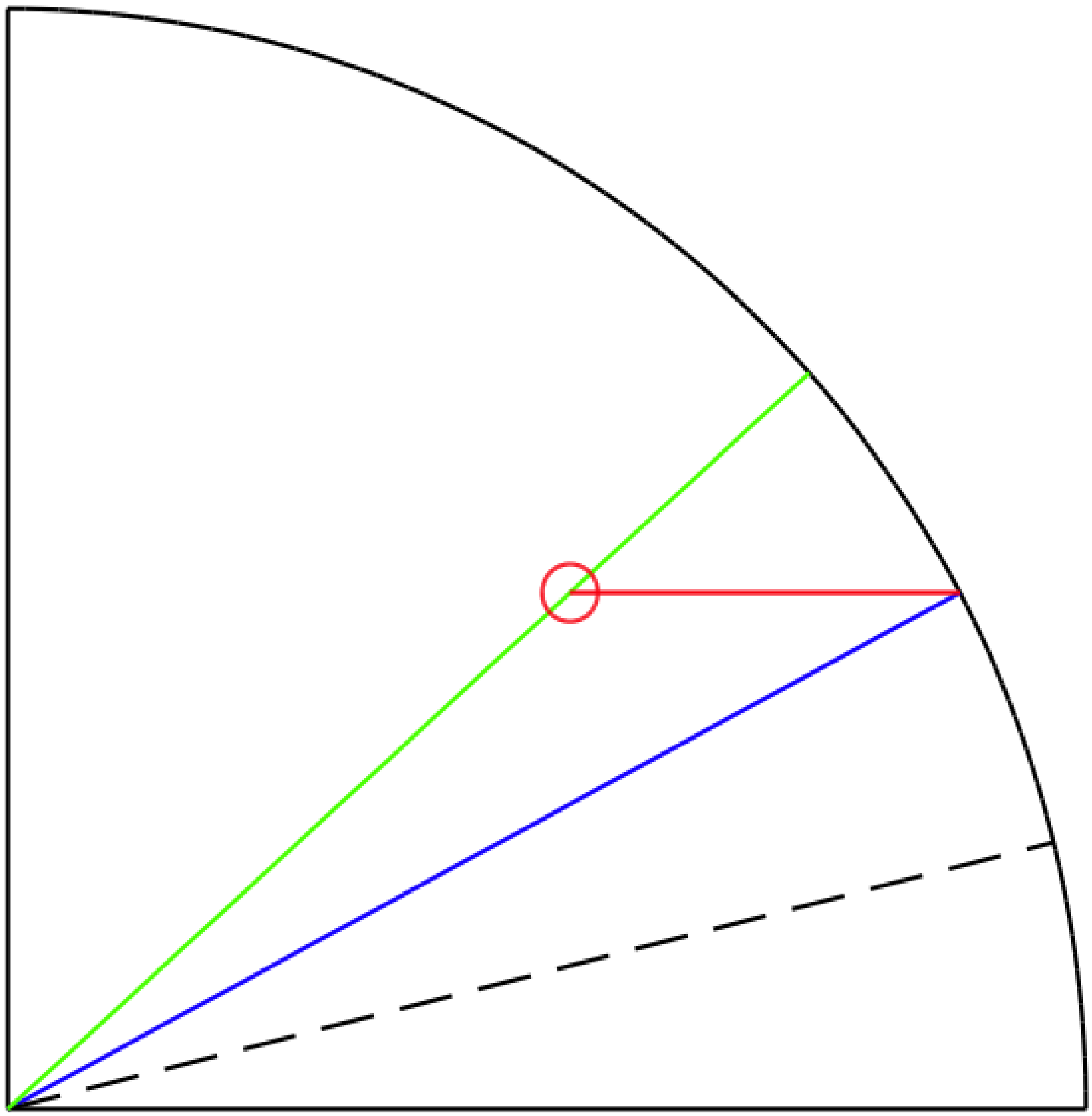
But simple solutions seem to invite other challenges (other than the naysayers and doubters) and these are more concerned with mathematical interpretation, expansion of this procedure, alternate solutions but most of all, satisfying an inquiring mind. With reference to the latter, being able to trisect and 5- sect angles lead to the solution of the three remaining “unsolved” ancient puzzles. And while this paper describes solutions to some age-old drafting puzzles its results have little to offer to the study of science or engineering. In a way the solutions might have been described briefly, as the answer to the trisection puzzle required only a single horizontal line. However, that line lead to a field of exercises in logic, methodology and discovery. And what might have been a simple paper became a thesis.

Having found a method that solves this ancient problem and the simplicity of its solution is puzzling, as I find it odd that this drafting challenge goes back to the time when Greek mathematicians and draftsmen explored the relationship of geometrical configurations, developed a number of principles and equations, angular and circular, and yet, the geometric problem of trisecting an angle, using only a compass and an unmarked straight edge was not solved? A mystery indeed, because the required tools and the procedures presented here very likely were used to develop the basic Sine trigonometric function and its cyclic waveform. This waveform- readily plotted using the same simple tools will be shown. Moreover, the methods used to solve this trisection puzzle and their results are used to solve the three remaining ancient puzzles: the Nine Sided Polygon, more notably - Squaring the Circle, and Doubling the Cube.

Prelude: As all of the puzzle solutions presented start by drawing the following basic diagram, it seemed best for the reader to become familiar with the terms used and the work area in which the solutions are to be drawn. The base circle radius (R) is most important because all solutions are worked out in the first quadrant of that selected circle. However, in the trisection solution a number of horizontal and vertical lines must be drawn, each from a single point. To ensure that the lines are accurately drawn, the full base circle is required. Methods of construction are shown later in a series of diagrams. The superimposed X-Y coordinate system shown in the first diagram is used for plotting or for calculating values.



The Trisection Solution: The model diagram shown in Fig. 1 is used for all sectioning procedures and is simply the first quadrant of an arbitrary base circle with a trisected angle. The angle is constructed by drawing an unknown angle, divide it in two then adding 1/2 of that angle to form the trisected angle shown - or by simply marking off three equal spaces along the base circle.



Step One Fig. 1

At this point by drawing a horizontal line between the second and third angular lines so that they intersect as shown, you have an intersecting point that is **singularly** characteristic of this trisected angle and this in fact, is the key to the **Solution** of the trisection puzzle. Repeating this procedure at angular intervals up to 90 deg. - Fig. 2, eventually gives us a series of intersecting points. Connecting these points as shown in Fig. 3 results in a 90 deg., arc-like curve within the base circle. At this point we have constructed a Trisection Curve: the completed solution to the Ancient puzzle!

Note that the curve does not stand alone - in order to trisect an angle, it must be paired with its original base circle. Fig. 4 illustrates how this curve is used to trisect two different angles with both showing excellent results. Note that these points are at exactly 2/3 of each unknown angle. Dividing the remaining portions of these angles and we complete the trisection of both angles. This curve is indeed the “Magic Circle” Trisectors have been trying to find for 2500 years. Also, we will show this procedure used to **5-sect** angles, as it can for even higher, prime divisional sections. The sectioning of prime numbered angles may prove useful in the field of Vector Analysis (perhaps my next assignment).

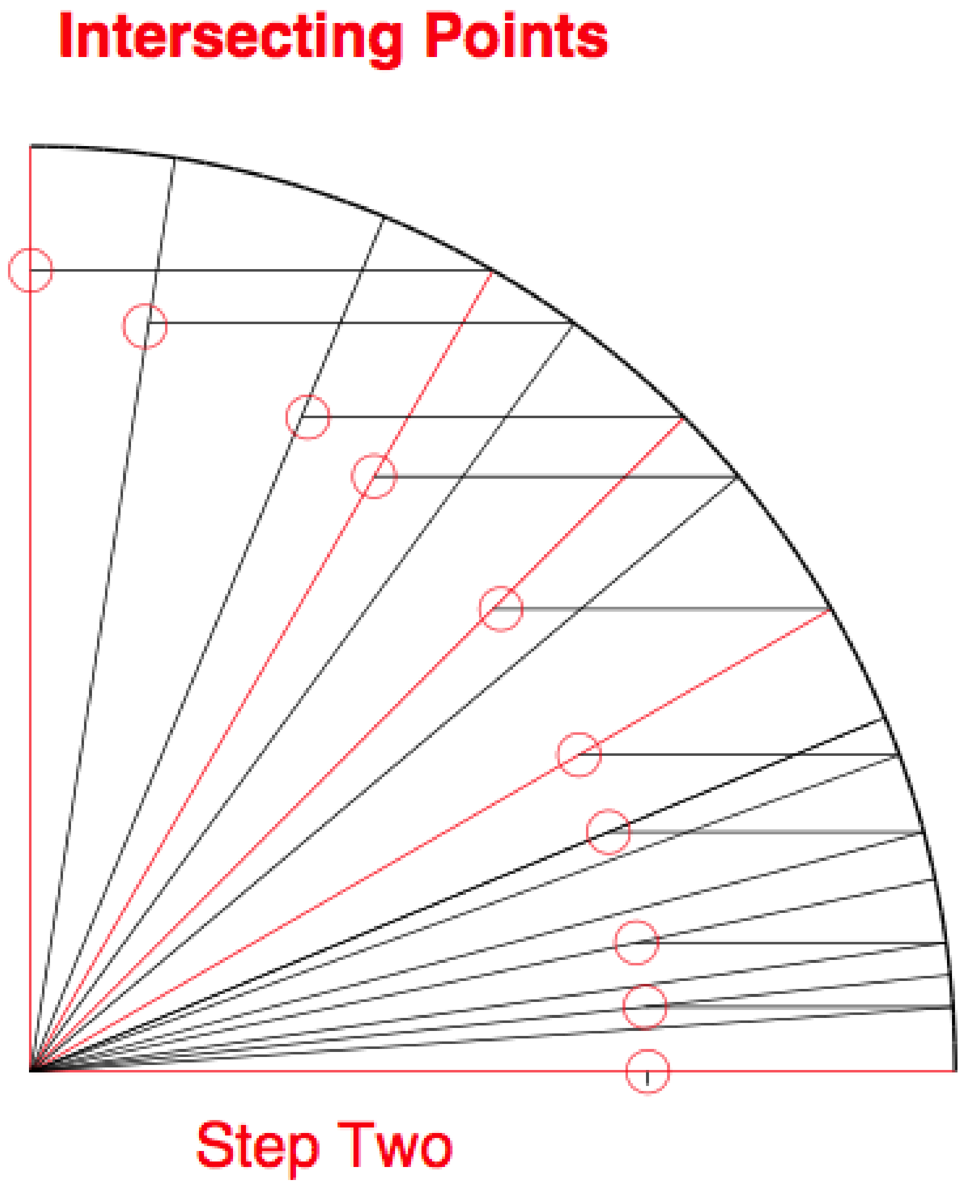
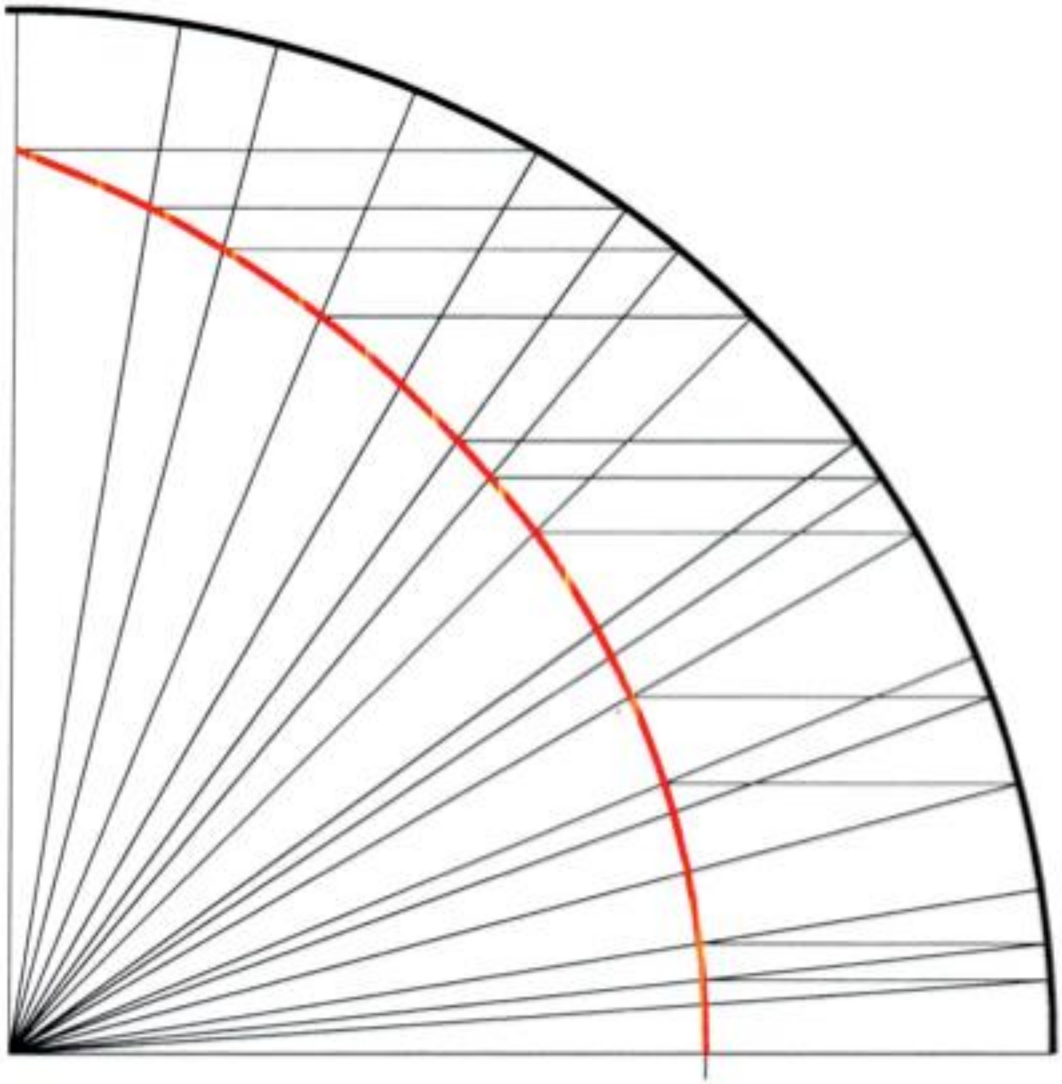


Fig. 2

Trisection Curve



Step 3

Fig. 3

Trisection of two angles

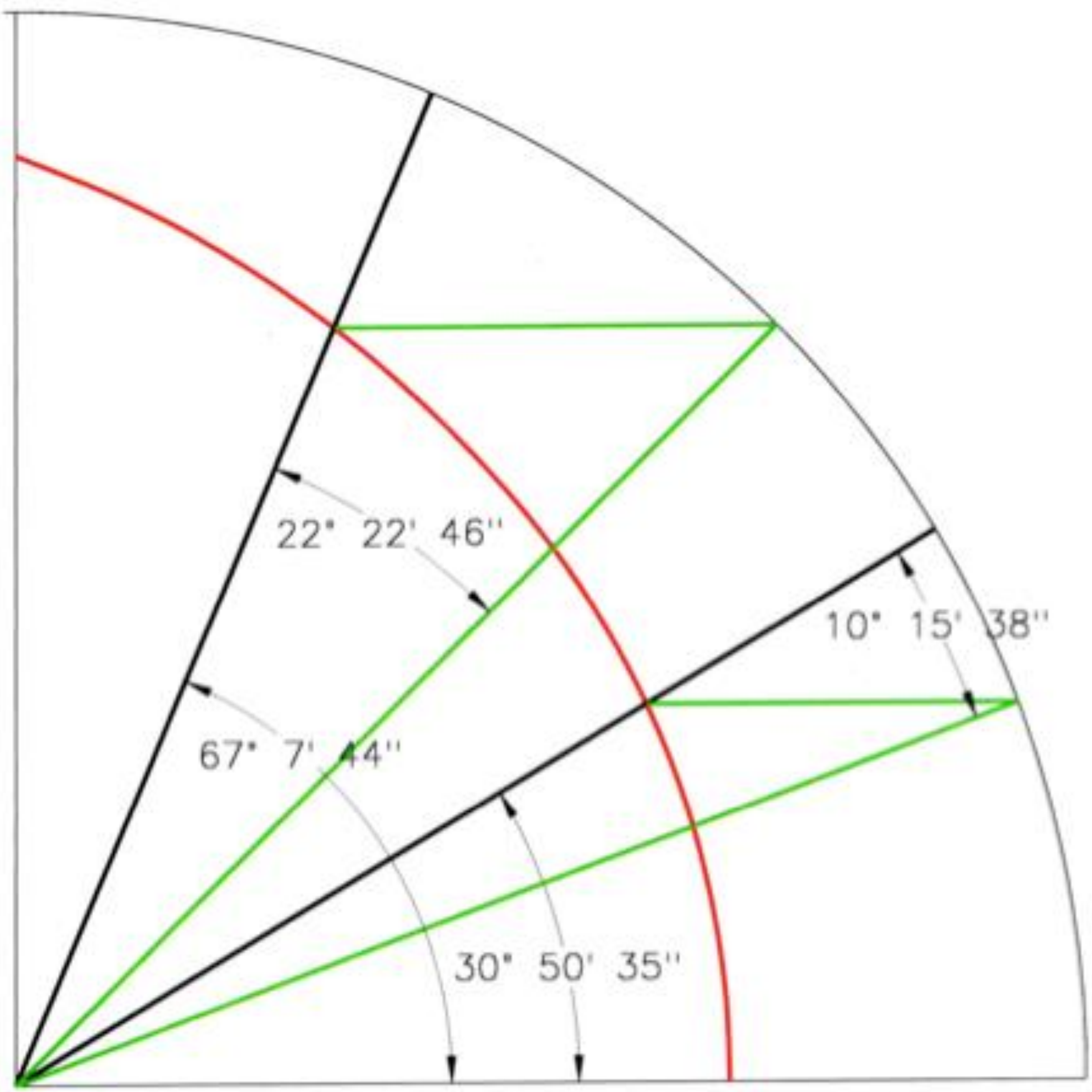


Fig. 4

However, a sharp eye will note that a horizontal line cannot be drawn from a single point. The following second diagram (Fig. 5) shows how the key horizontal lines are drawn for the trisection solution. Line One is drawn from the original unknown angle at the base circle, to the added angle. This intersecting point helps form the trisection curve. Line Two is the trisection solution line - drawn from the intersecting point shown on the trisection curve, to the base circle; trisecting the original angle. Note the angular red line that locates the second point for horizontal line TWO.

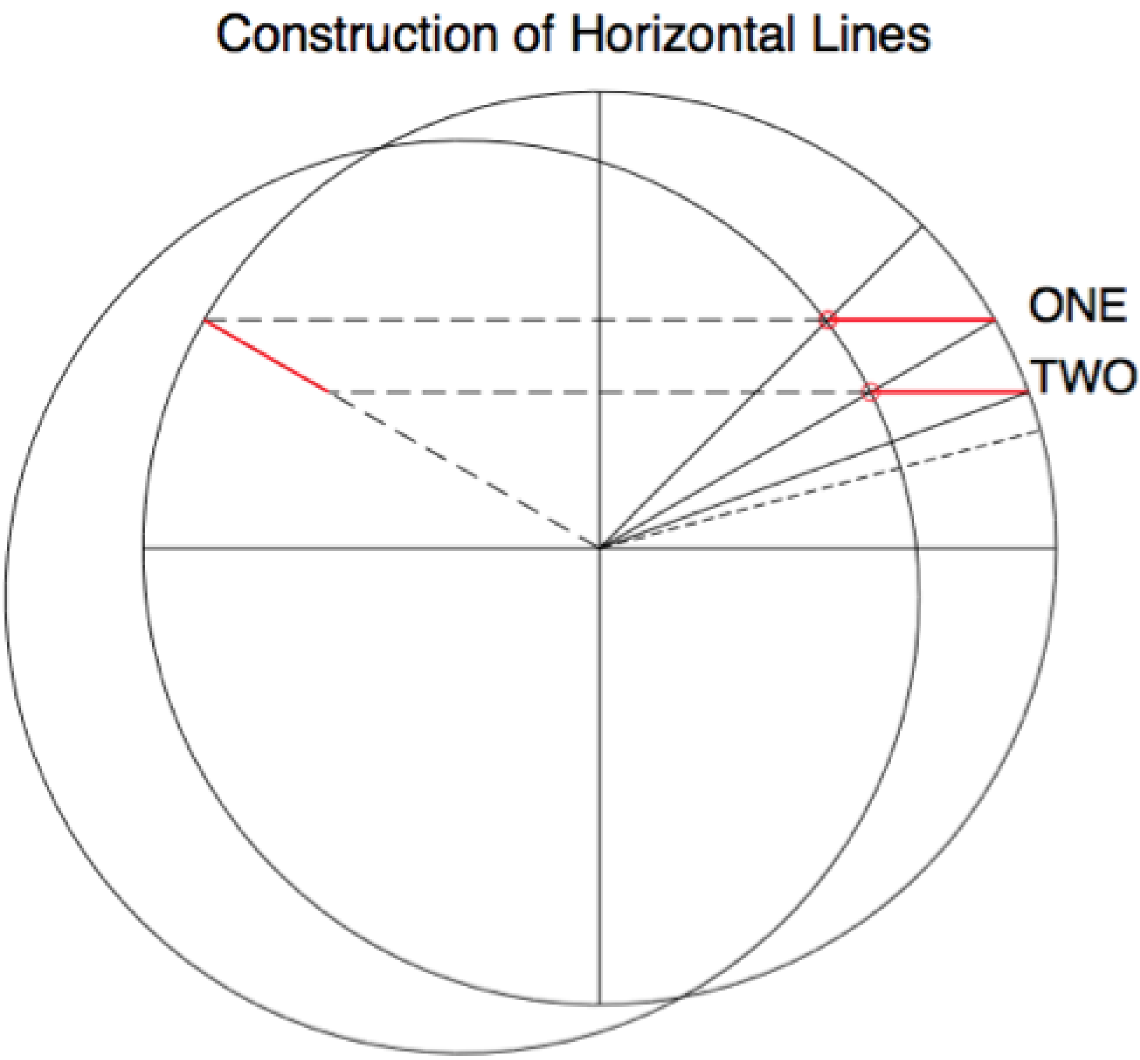


Fig. 5

Further Constructions: For those not familiar with some of the drafting techniques used to construct the base line and circle, and the angular, vertical and horizontal lines, the following drawings illustrate how these may be drawn using only a straight edge and a compass. Indicated in the following drawings are four procedural steps:

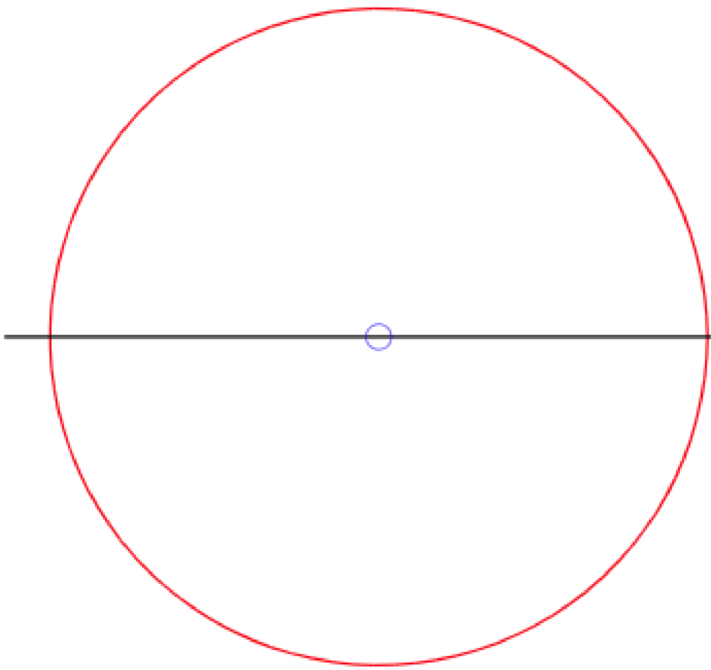
Step 1. Draw a horizontal line. Within this line draw a circle.

Step 2. Draw three circles of equal size as shown with centers on the base line as indicated. The intersecting circles provide points used to draw the vertical lines shown.

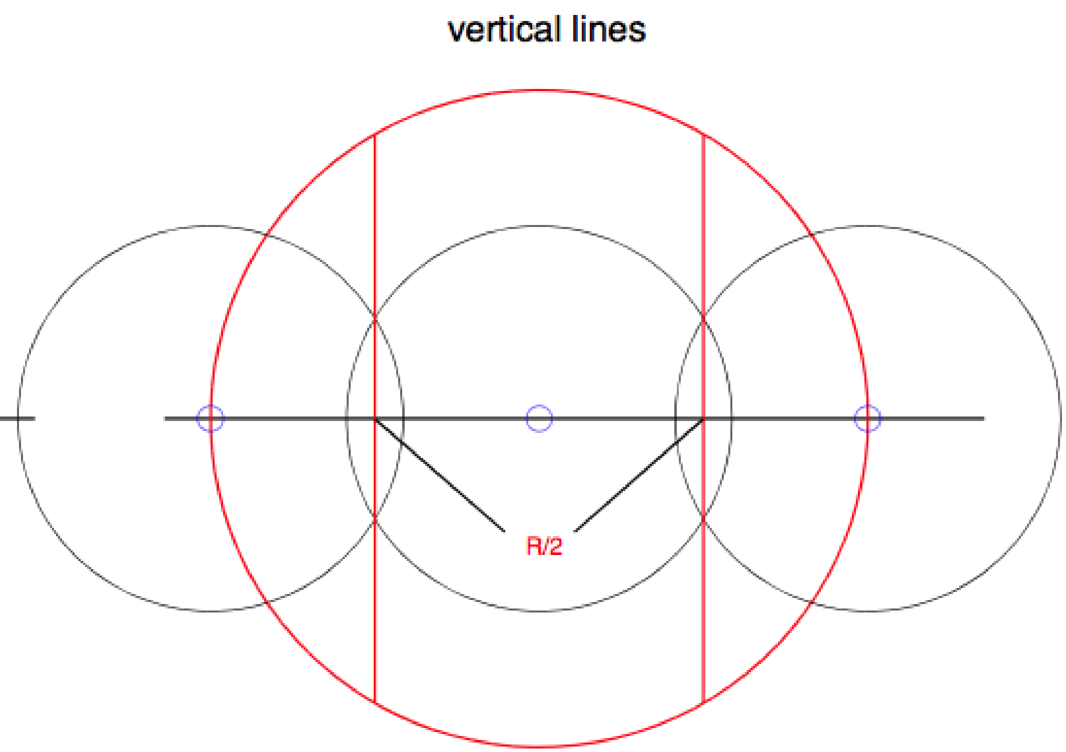
Step 3. Locate two equal size circles at the vertical lines intersecting points and draw a center vertical line. Note this line forms the very important 1st quadrant.

Step 4. Shows how horizontal lines are drawn using opposite points on a circle. Note also that bisecting the radius with the vertical lines (step3) also locates the 60deg. angle. The dotted lines are an arbitrary distance measured from the base line to the circle on each side and we have two points for a horizontal line.

Base Circle
Construction



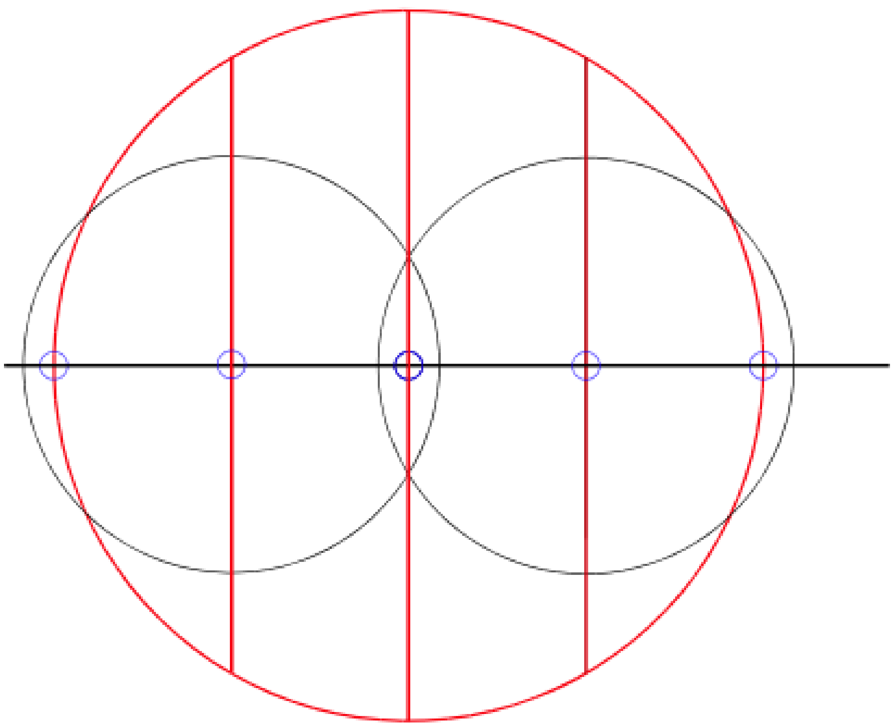
Step One



Step two

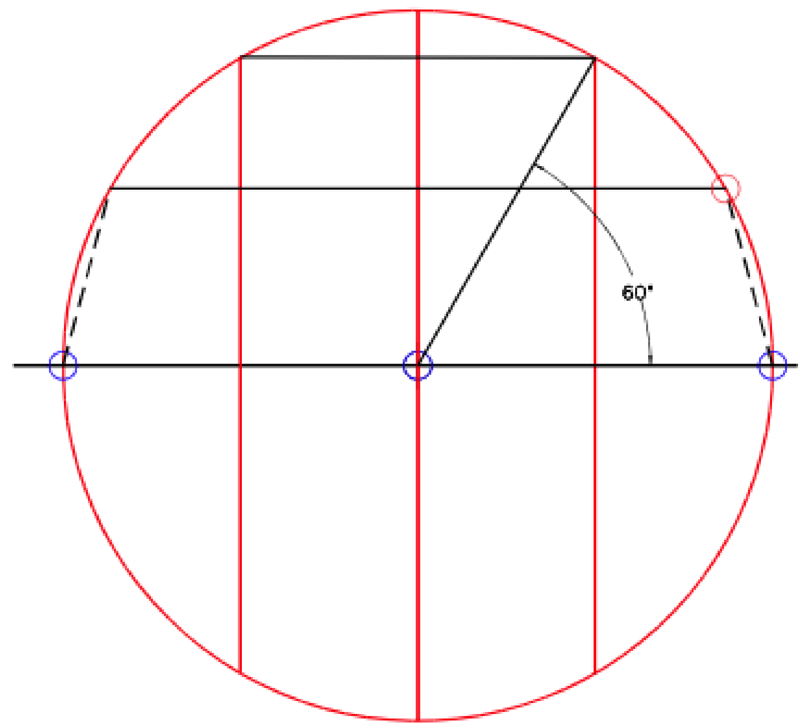
Base Circle
Construction

quadrant line



Step 3

Horizontal lines



Step 4

Trisecting Angles Greater than 90 deg.: It is important to note that the Trisection curve does not extend beyond 90 deg. of the first quadrant of the base circle. So how does one trisect angles greater than 90 deg.? For angles greater (or less) than 90 deg., Fig. 6 illustrates how they can be trisected readily, either by quartering the angle, so that the last quarter lies in the first quadrant, or by simply marking off three equal spaces with a remainder that can be trisected. Then one simply adds a segment to the top section of the original angle and you have trisected the larger angle!

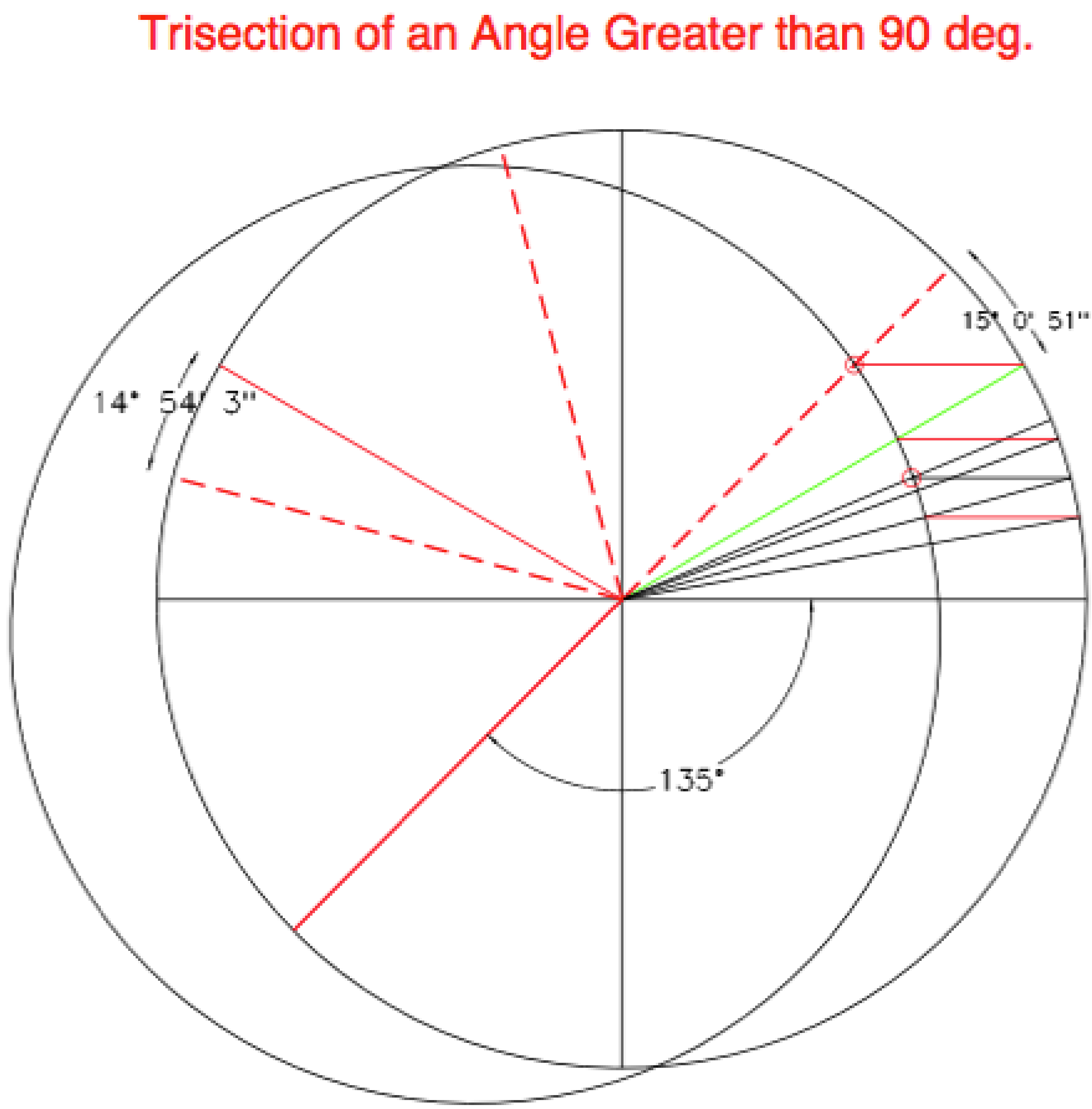


Fig. 6

5-secting an Angle: Here the process is the same as in Fig. 1, except we start with a 5-sected angle. We do this by drawing an unknown angle, and adding ¼ of that angle to get a 5-sected angle. We then draw a horizontal line between the fourth and fifth angular line and locate the intersecting point. Fig. 7 shows a 5-sected angle (using a segmented procedure to be described). This figure is used to solve the problems "Squaring the Circle" and "Doubling the Cube

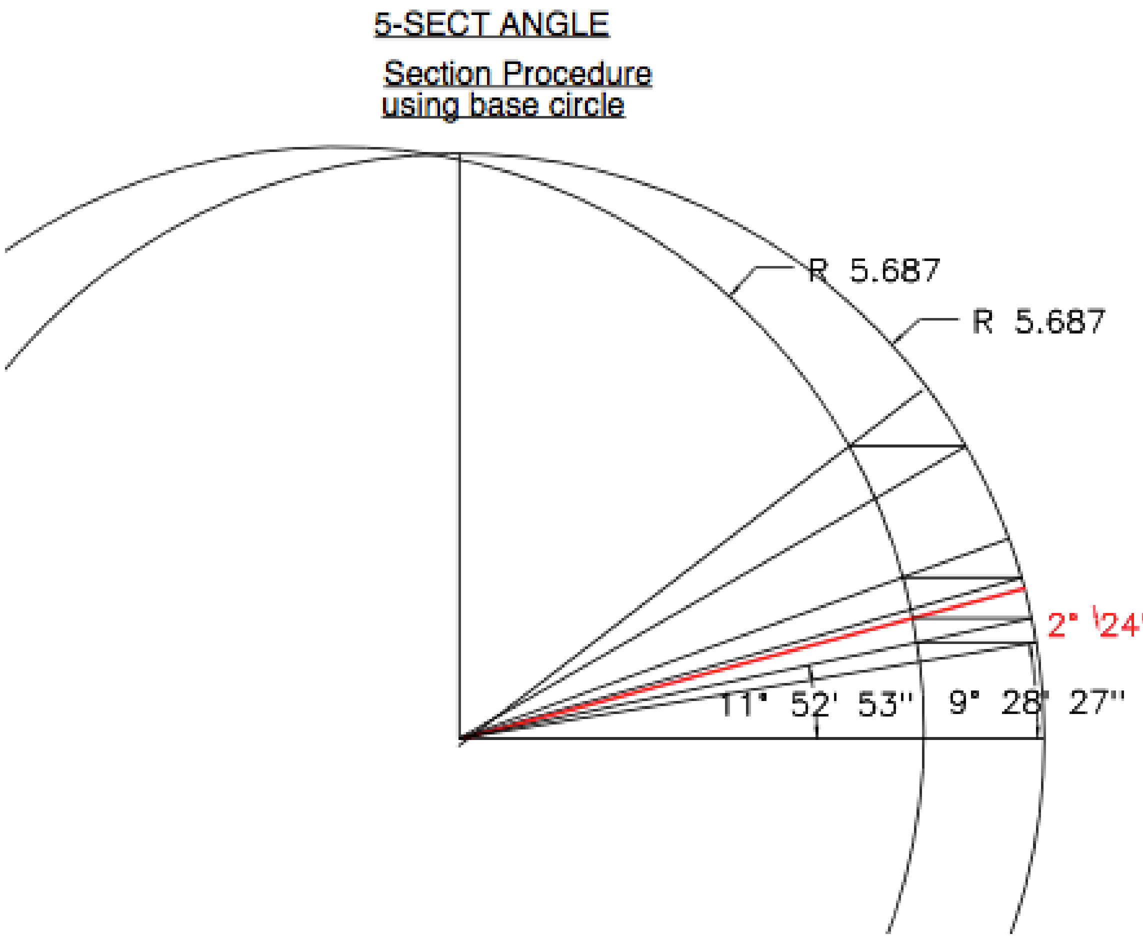


Fig.7

The Sine Wave: Some might deem it tedious to construct a trisection curve merely to trisect one angle and dismiss it as an impractical solution- although there is no other. To counter such objections e.g. just a series of trisected angles, one should look at the construction of the sine wave shown in Fig. 8 shows an incremental technique where the magic curve equals the base circle for any angle and the value of Y at each angle of intersection is at the base circle. The values of Y are then carried over to a different co-ordinate system where they are plotted against the same intersecting angle. In like manner, repeating this procedure to get the number of horizontal lines needed to plot the waveform can also be tedious, but who can question its accuracy or use if it is constructed meticulously using only a straight edge and a compass. After all -it is the basis for all trigonometric functions.

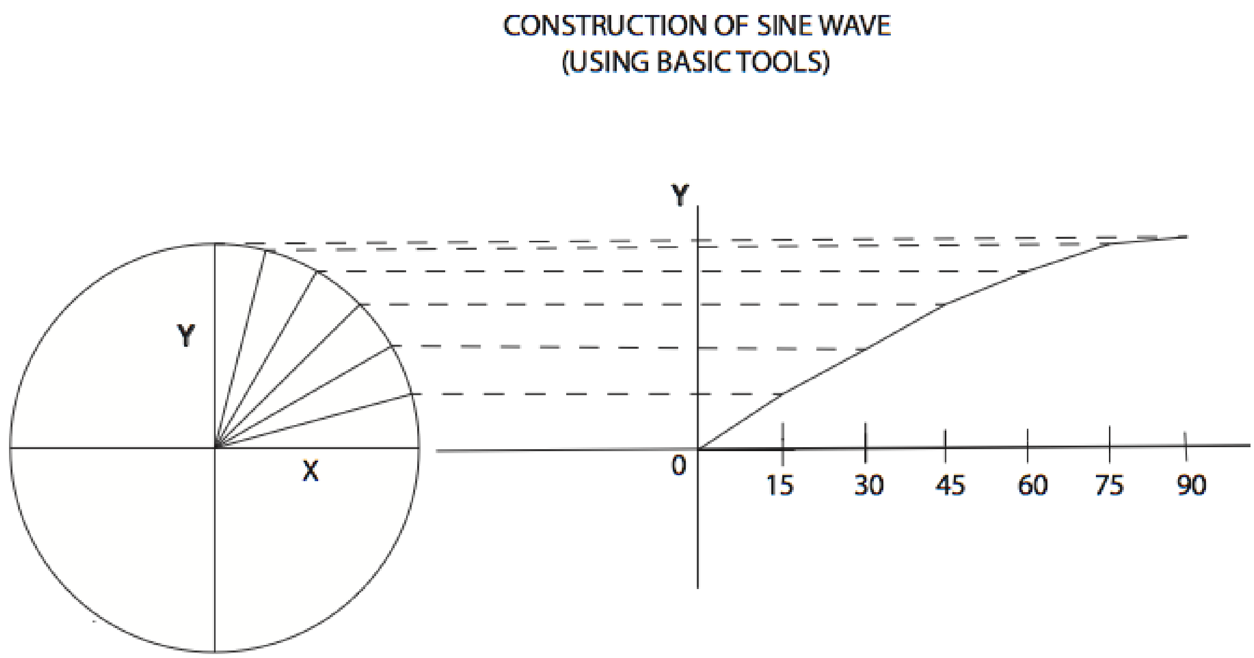


Fig. 8

Mathematical Solution: During the past centuries and in more recent times, many have tried to find a mathematical solution to this problem or to prove why it could never be solved.

Sadly, to say Wikipedia is filled with ever growing comments from mathematicians who can prove that this or any of the other problems cannot be solved. Realizing that although this manual method of plotting the intersecting points to form the Trisection curve is obviously correct and the only solution, it still seemed necessary to further verify the accuracy of the process. So for those mathematicians who dwell in the never-never land of negativity, I present the following: The graph Fig. 9 shows a mathematically plotted base circle, a trisection curve and the equations used to calculate the values of (X and Y). Also, it shows how the graph is used to trisect two angles. The last plot (Fig. 10) shows both the Trisection and 5- section curves, along with their common base circle. Here a single angular line is both trisected and 5-sected. Bear in mind that the mathematical equations used are based on manually constructed models.

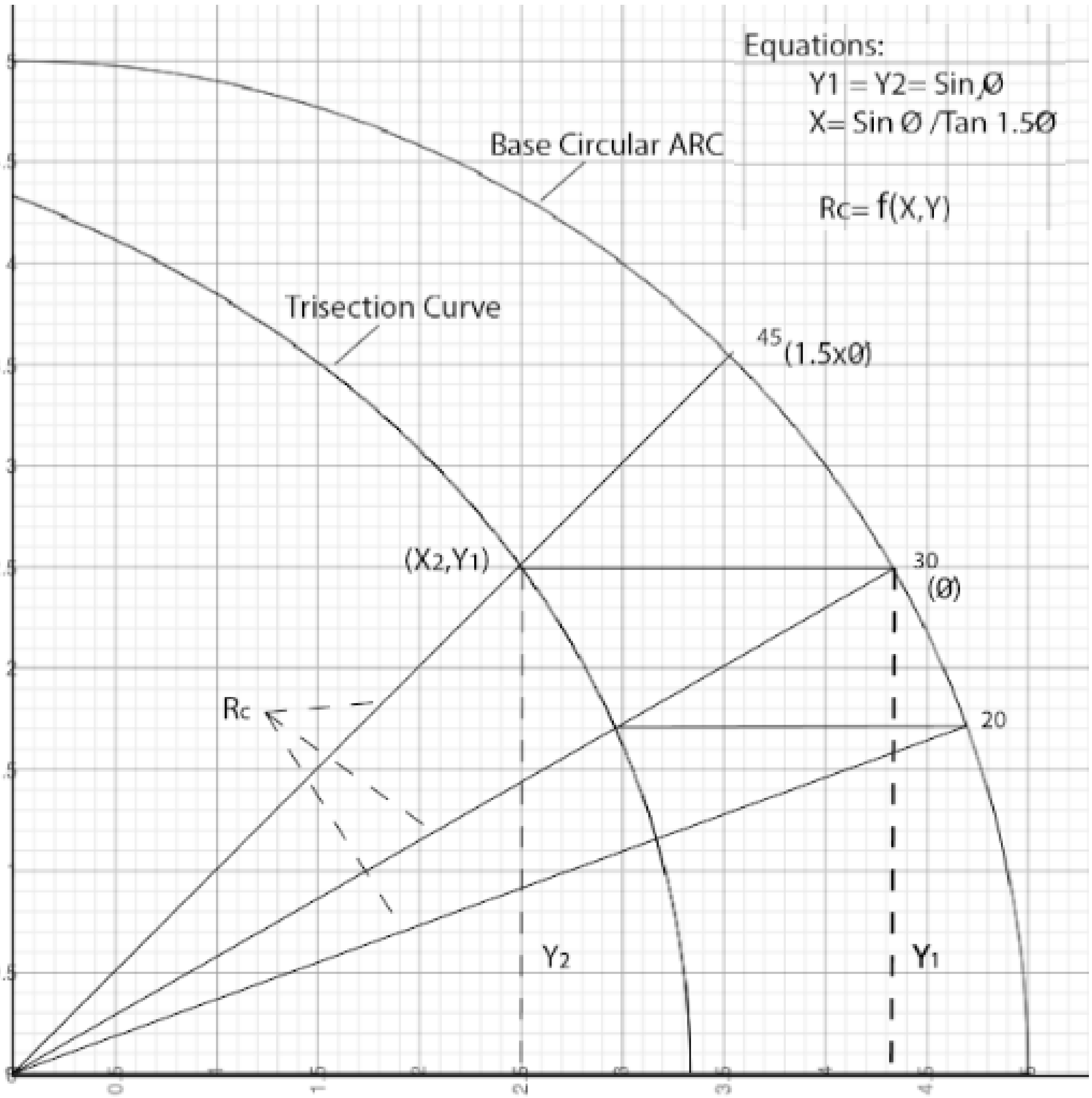

Equations:
Y1 = Y2= Sin Ø
X= Sin Ø /Tan 1.5Ø
Rc= f(X,Y)
Base Circular ARC
Trisection Curve
45 (1.5xØ)
(X2,Y1)
30 (Ø)
20
Rc
Y2
Y1

Fig. 9

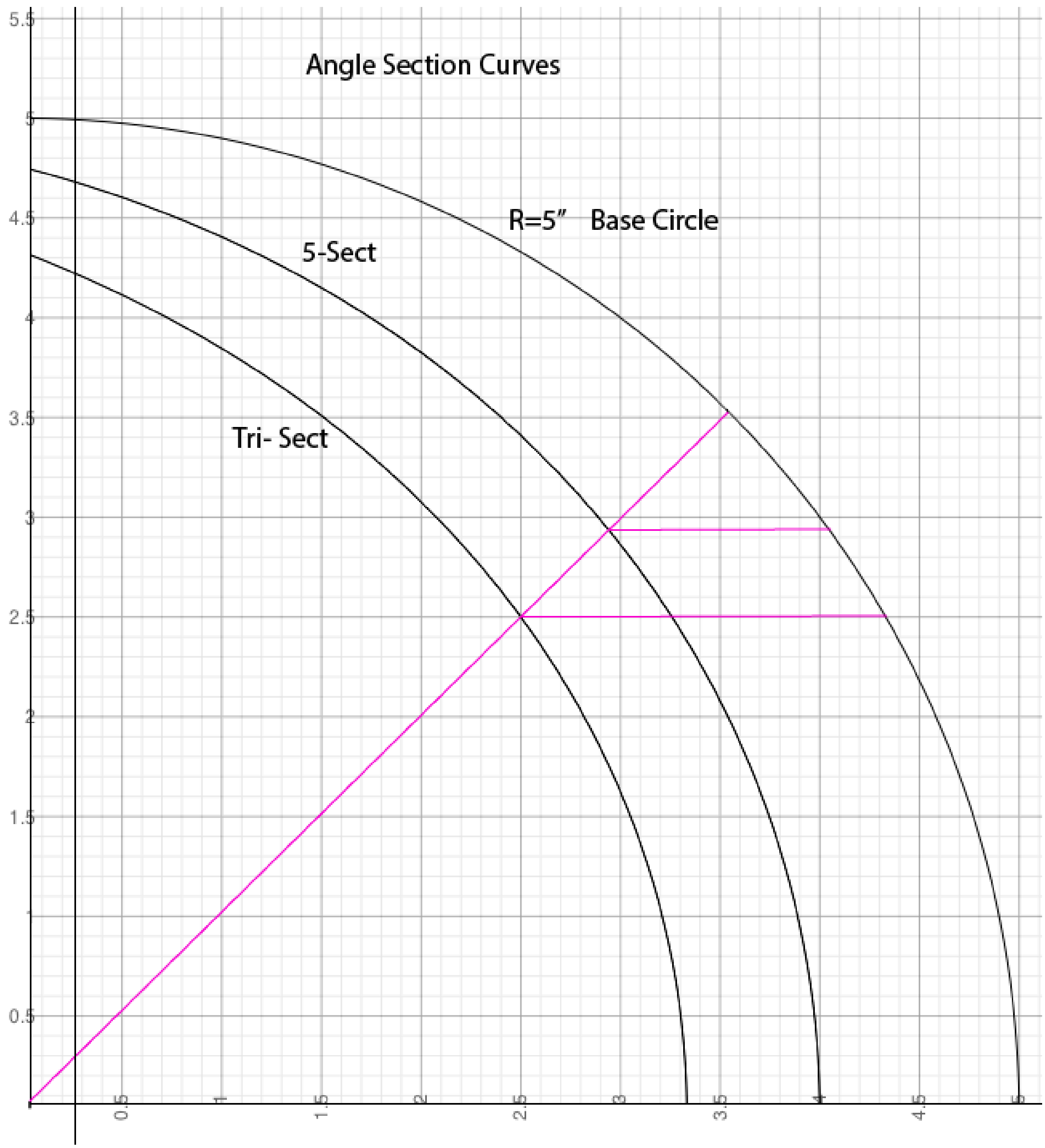

Angle Section Curves
R=5"   Base Circle
5-Sect
Tri- Sect
5.5
5
4.5
4
3.5
3
2.5
2
1.5
0.5
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Fig.10

The Magic Circle: Initially, after plotting a number of trisecting points, it appeared that the shape of what turned out to be the trisection curve closely matched the curvature of a displaced 60 deg. portion of the base circle. and I believed I had my Magic Circle. However, in drawing a second circle having the same base radius and shifting its center to a point where the base circle closely matched the trisection curve at several points, there were some minor but apparently unacceptable errors. Interestingly enough, by further shifting the center of this magic center a small distance the trisection errors were reduced in one area but increased in another! Although I knew I already had an exact solution subject only to drafting errors, the idea of the base circle properly located would simplify the procedure considerable. But what degree of error would be acceptable?

At this point I decided to contact Jim Loy at Wikipedia, sending him a step-by-step procedure. His initial response was favorable, writing that my approach was different than any he had seen before and that he was able to get very accurate results for small angles (less than 45 deg.) but the accuracy falls off at higher angles. The problem here was that we were looking for a single, fixed, centered "Magic" circle, one, in which with two lines drawn to this circle would trisect any angle. However, as indicated in graphs Fig. 11 & 12, by shifting its center the base circle can trisect smaller ranges of angles with acceptable accuracy within the first quadrant. Having sent these to Jim Loy, he responded, saying that to keep shifting the base circle, however accurate, would be just "fiddling around," and therefore was not an acceptable solution.

Also that trisected errors above 0.10 deg. would not be acceptable. Yet as shown in these figures with shifted base circler centers, both show regions of trisections that are exact.

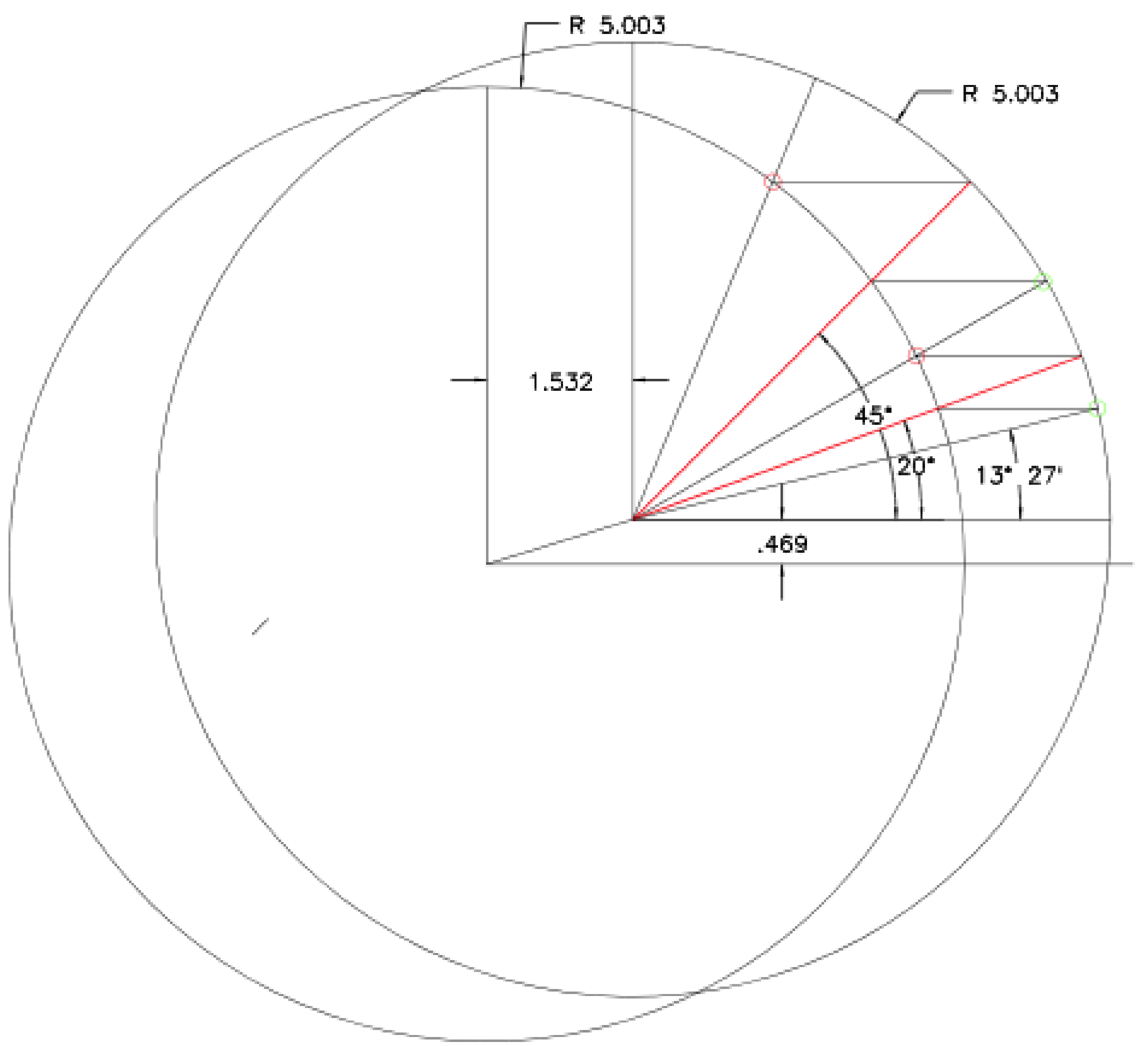


Fig. 10

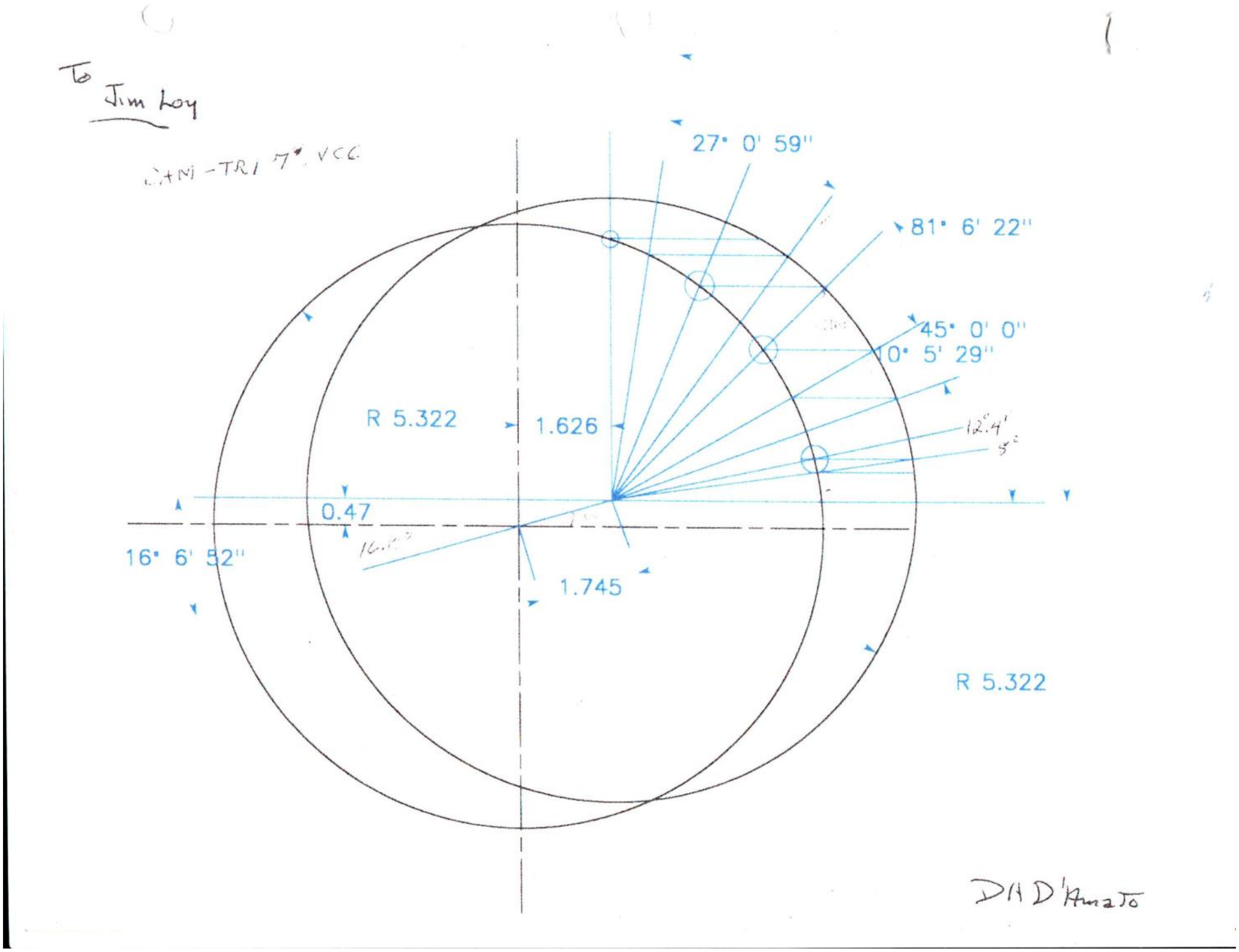

To Jim Loy
27° 0' 59"
81° 6' 22"
45° 0' 0"
0° 5' 29"
R 5.322
1.626
0.47
16° 6' 52"
1.745
R 5.322
DA D'Amato

Fig. 11

Erroneously convinced that I might find the perfect center position for the duplicate base circle (obviously following the path of most Trisectors), I decided to perform an error analysis using mathematical programs and some simple equations to list and plot every intersecting point from 1 to 90 deg. on the Trisection curve and corresponding angular points on shifted centers of the base circle. One data print out is shown as well as two error profiles in the following pages Fig. 12 & 13. The results did show less than 0.5% peak radial errors, but more importantly relatively broad bands of highly accurate results.

This suggested a second - but highly useful - trisection method whereby a floating base circle concept might be used to form accurate, segmental regions for trisecting a single angle.

DEGREES	radians	SIN	Y x5	tan(1.5*deg)	X
0	0	0	0	0	
2	0.03490659	0.0348995	0.17449748	0.05240778	3.3296103
3	0.05235988	0.05233596	0.26167978	0.07870171	3.3249569
4	0.06981317	0.06975647	0.34878237	0.10510424	3.3184426
5	0.08726646	0.08715574	0.43577871	0.1316525	3.310068
6	0.10471976	0.10452846	0.52264232	0.15838444	3.2998337
7	0.12217305	0.12186934	0.60934672	0.18533904	3.2877407
8	0.13962634	0.1391731	0.6958655	0.21255656	3.2737898
9	0.15707963	0.15643447	0.78217233	0.24007876	3.2579822
10	0.17453293	0.17364818	0.86824089	0.26794919	3.2403191
11	0.19198622	0.190809	0.95404498	0.29621349	3.2208019
12	0.20943951	0.20791169	1.03955845	0.3249197	3.1994319
13	0.2268928	0.22495105	1.12475527	0.35411857	3.1762109
14	0.2443461	0.2419219	1.20960948	0.38386404	3.1511404
15	0.26179939	0.25881905	1.29409523	0.41421356	3.1242222
16	0.27925268	0.27563736	1.37818678	0.44522869	3.0954582
17	0.29670597	0.2923717	1.46185852	0.47697553	3.0648501
18	0.31415927	0.30901699	1.54508497	0.50952545	3.0324
19	0.33161256	0.32556815	1.62784077	0.5429557	2.9981097
20	0.34906585	0.34202014	1.71010072	0.57735027	2.9619813
21	0.36651914	0.35836795	1.79183975	0.61280079	2.9240167
22	0.38397244	0.37460659	1.87303297	0.64940759	2.8842178
23	0.40142573	0.39073113	1.95365564	0.68728096	2.8425866
24	0.41887902	0.40673664	2.03368322	0.72654253	2.7991248
25	0.43633231	0.42261826	2.11309131	0.76732699	2.7538342
26	0.45378561	0.43837115	2.19185573	0.80978403	2.7067164
27	0.4712389	0.4539905	2.2699525	0.85408069	2.6577729
28	0.48869219	0.46947156	2.34735781	0.90040404	2.607005
29	0.50614548	0.48480962	2.4240481	0.94896457	2.5544137
30	0.52359878	0.5	2.5	1	2.5
31	0.54105207	0.51503807	2.57519037	1.05378013	2.4437644
32	0.55850536	0.52991926	2.64959632	1.1[illegible]61251	2.385707[illegible]
33	0.57595865	0.54463904	2.72319518	1.17084957	2.3258284
34	0.59341195	0.5591929	2.79596452	1.23489716	2.2641274
35	0.61086524	0.57357644	2.86788218	1.30322537	2.2006034
36	0.62831853	0.58778525	2.93892626	1.37638192	2.1352549
37	0.64577182	0.60181502	3.00907512	1.45500903	2.06808
38	0.66322512	0.61566148	3.07830738	1.53986496	1.9990762
39	0.68067841	0.62932039	3.14660196	1.63185169	1.9282402
40	0.6981317	0.64278761	3.21393805	1.73205081	1.855568
41	0.71558499	0.65605903	3.28029514	1.84177089	1.7810549
42	0.73303829	0.66913061	3.34565303	1.96261051	1.7046954
43	0.75049158	0.68199836	3.4099918	2.0965436	1.6264827
44	0.76794487	0.69465837	3.47329185	2.24603677	1.5464092
45	0.78539816	0.70710678	3.53553391	2.41421356	1.4644661
46	0.80285146	0.7193398	3.596699	2.60508906	1.3806434
47	0.82030475	0.7313537	3.65676851	2.82391289	1.2949296
48	0.83775804	0.74314483	3.71572413	3.07768354	1.207312
49	0.85521133	0.75470958	3.7735479	3.37594342	1.1177758
50	0.87266463	0.76604444	3.83022222	3.73205081	1.0263049
51	0.89011792	0.77714596	3.88572981	4.16529977	0.9328812
52	0.90757121	0.78801075	3.94005377	4.70463011	0.8374843
53	0.9250245	0.79863551	3.99317755	5.39551717	0.7400917
54	0.9424778	0.80901699	4.04508497	6.31375151	0.6406785
55	0.95993109	0.81915204	4.09576022	7.59575411	0.5392171
56	0.97738438	0.82903757	4.14518786	9.51436445	0.4356768
57	0.99483767	0.83867057	4.19335284	12.7062047	0.330024
58	1.01229097	0.8480481	4.24024048	19.0811367	0.2222216
59	1.02974426	0.8571673	4.2858365	38.1884593	0.1122286
60	1.04719755	0.8660254	4.33012702	1.6325E+16	
61	1.06465084	0.87461971	4.37309854		
62	1.08210414	0.88294759	4.41473796		
63	1.09955743	0.89100652	4.45503262		
64	1.11701072	0.89879405	4.49397023		
65	1.13446401	0.90630779	4.53153894		
66	1.15191731	0.91354546	4.56772729		
67	1.1693706	0.92050485	4.60252427		

Fig. 12

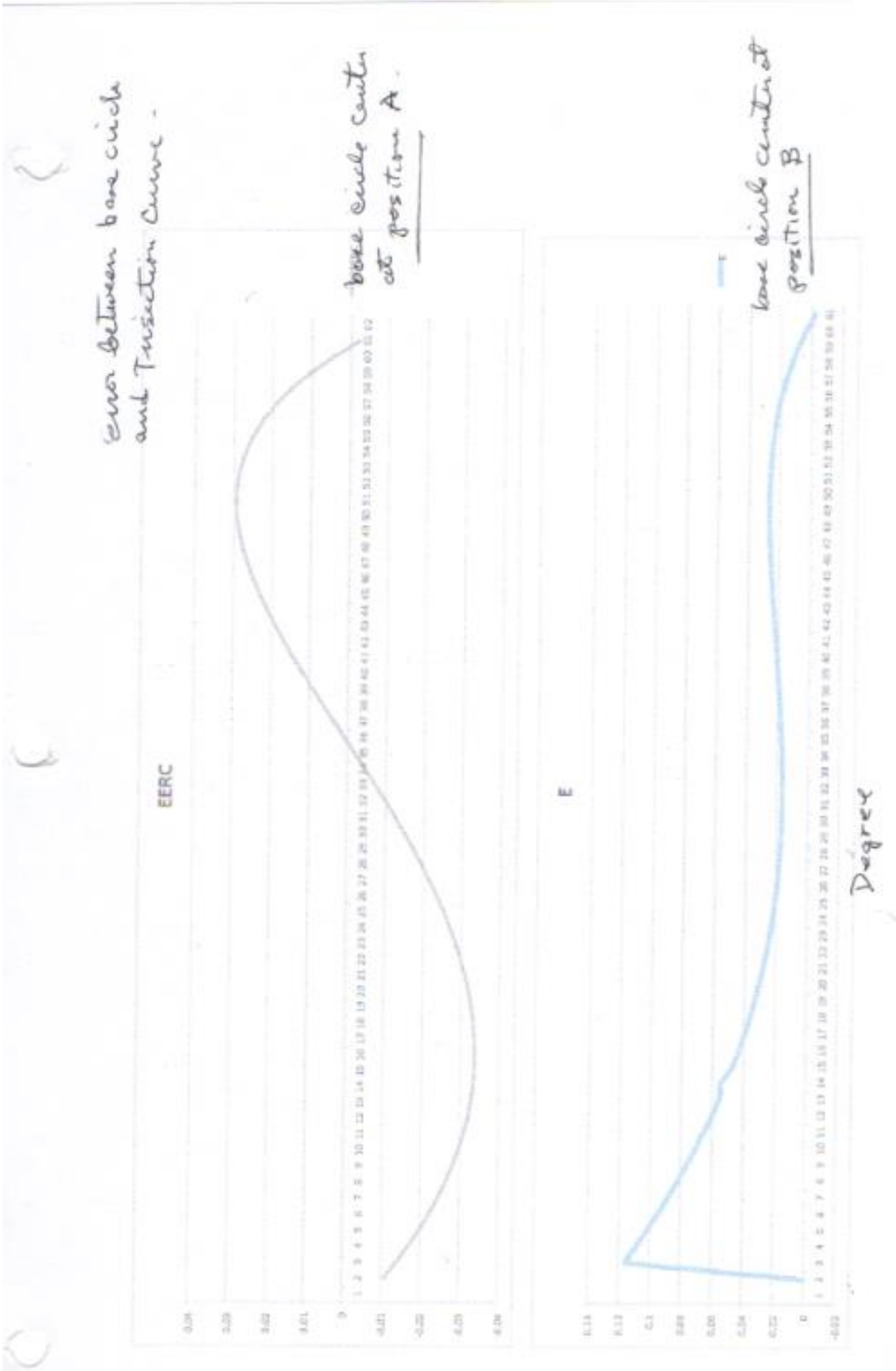

error between base circle and Trisection Curve.
EERC
base circle center at position A.
E
base circle center at position B
Degree

Fig. 13

Trisecting a Single Angle: This procedure to be described may prove to be the more acceptable solution to the trisection puzzle as it combines the use of the base circle and the trisection plotting process. As Fig. 13 indicates there are angular regions in the quadrant where the center of the base circle properly located gives near perfect results i.e. much less than .1 deg. However, the Trisection curve is an exact solution but it is time consuming to plot if one wants to trisect a single angle. Thus neither procedure is expedient in performing the original task; trisecting an unknown angle. But in the concept of incremental analysis, as noted above, there is a quicker and accurate solution if one uses a segmented procedure illustrated in Fig. 14. Here the base circle, used to form only a small section of the Trisection curve, floats. Note that in this procedure, the optimum base circle center is located for any angle using only two intersecting points and the base circle radius. Two examples are shown in Fig. 14. Note the difference in the center point positions of the base circle and the **exact results.**

In essence then, this procedure proves to be the ultimate solution: practical and accurate for sectioning any angle.

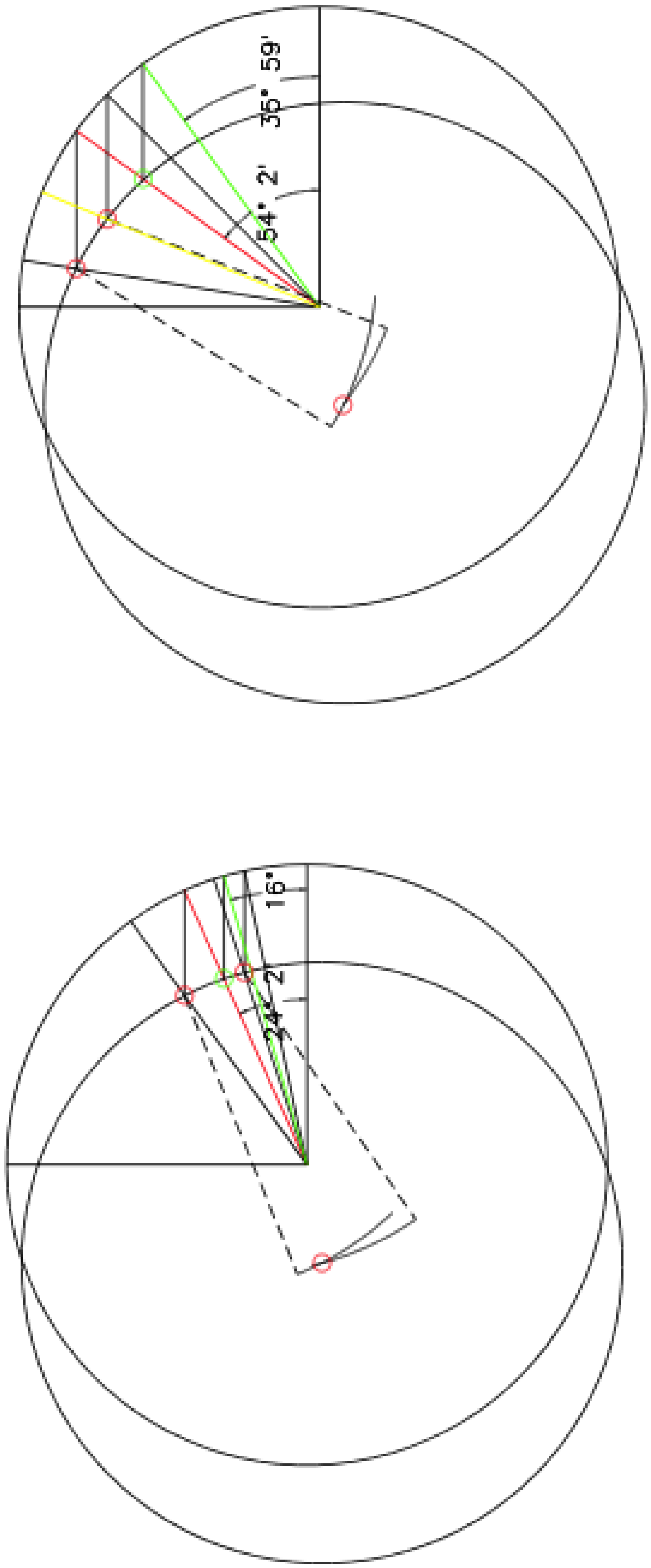


Fig. 14

The Magic Quadrant: The first quadrant of a circle is useful for defining trigonometric functions and for determining the angles used to construct the nine sided polygon and to solve the problems of squaring the circle and doubling the cube. More importantly it is the center point in the construction and understanding of the trisection process. For example, let's look at a trisection that was probably done 2,000 years ago. Note the drawings in Fig. 15, a simple quadrant of a circle with radius R. Now if vertical and horizontal lines are drawn from the mid-point of the horizontal radius and the vertical radius, respectively, to the circle, you have located the 30 and 60 deg. angles and have done the "impossible"; trisected a 90 deg. angle using a circle and lines. Add the 45 deg. line as shown and divide the 30 deg. angle and you have two trisected angles (45 & 90). Now let us add to this quadrant, using my procedure and introduce the magic horizontal lines. If we use the same quadrant and bisect the angles indicated in Fig. 3, add the halve angles and the horizontal lines to locate the intersecting points, then as shown, we have a total of four intersecting points which must lie on the Trisection curve. Note also, that we have three trisected angles. We now see that an angular line, drawn from the apex of the quadrant to the base circle, intersects the magic arc (when fully constructed) and is trisected by simply drawing a horizontal line from the intersecting point to the base circle. But although we have already constructed a magic arc, a curve that was based on unknown angles, the completion of this trisection curve with its known angles, is required for the solution to the three other "insolvable puzzles".

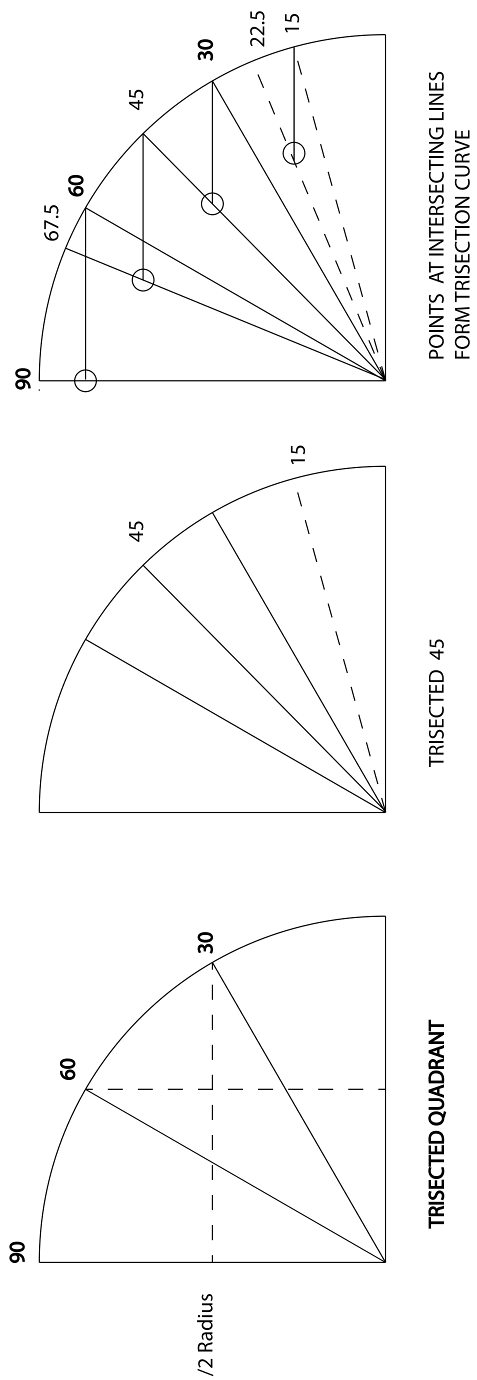

90
67.5
60
45
30
22.5
15
POINTS AT INTERSECTING LINES
FORM TRISECTION CURVE
45
15
TRISECTED 45
90
60
30
1/2 Radius
TRISECTED QUADRANT

Fig. 15

The Nonagon: The nine-sided polygon is another ancient problem thought to be impossible to construct using only the simple tools previously described. As each of the nine sides of a nonagon is a 40 deg. angle, relative to its center, one must be able to trisect the known angles of 30 or 60 deg. Fig. 16 shows the constructed solution and the segmented procedure used to locate the center of the base circle using the known 30 and 45 deg. angles and the trisection of the 30 deg. angle. The 20 deg. angle was doubled and the 40 deg. angle was marked off around the base circle to complete the construction. The trisection of 20 deg. is simply to show the accuracy of using the base circle in this region.

NINE SIDED POLYGON

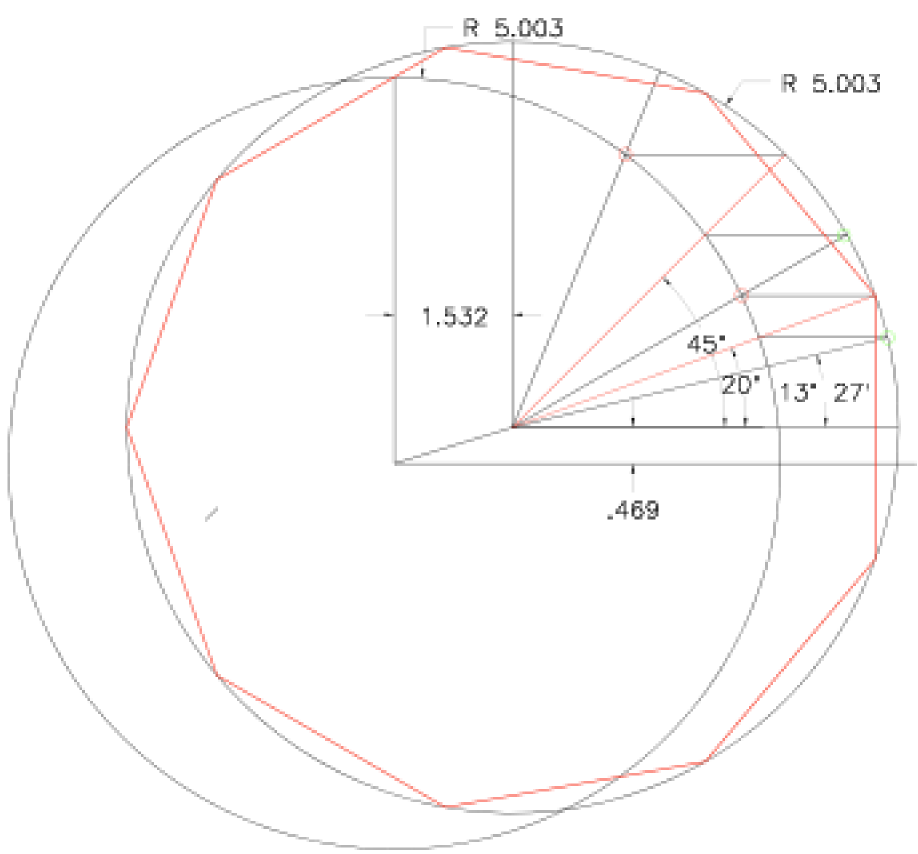


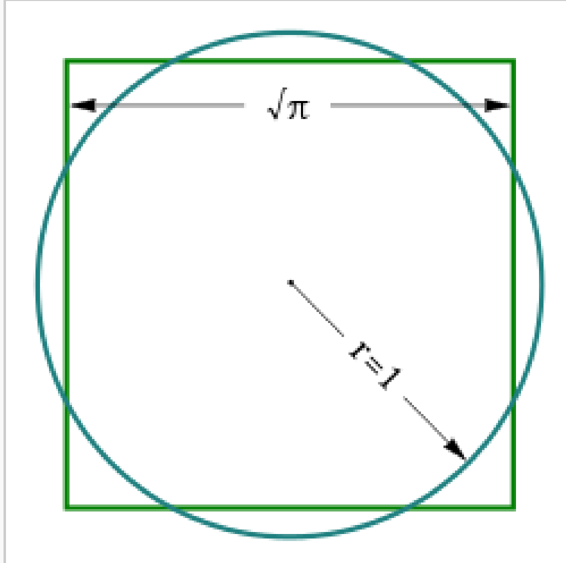
Fig. 16

Squaring the circle

From Wikipedia, the free encyclopedia

For other uses, see Square the Circle.

Squaring the circle is a problem proposed by ancient geometers. It is the challenge of constructing a square with the same area as a given circle by using only a finite number of steps with compass and straightedge. More abstractly and more precisely, it may be taken to ask whether specified axioms of Euclidean geometry concerning the existence of lines and circles entail the existence of such a square.



Squaring the circle: the areas of this square and this circle are both equal to π. In 1882, it was proven that this figure cannot be constructed in a finite number of steps with an idealized compass and straightedge.

In 1882, the task was proven to be impossible, as a consequence of the Lindemann–Weierstrass theorem which proves that pi (π) is a transcendental, rather than an algebraic irrational number; that is, it is not the root of any polynomial with rational coefficients. It had been known for some decades before then that the construction would be impossible if pi were transcendental, but pi was not proven transcendental until 1882. Approximate squaring to any given non-perfect accuracy, in contrast, is possible in a finite number of steps, since there are rational numbers arbitrarily close to π.

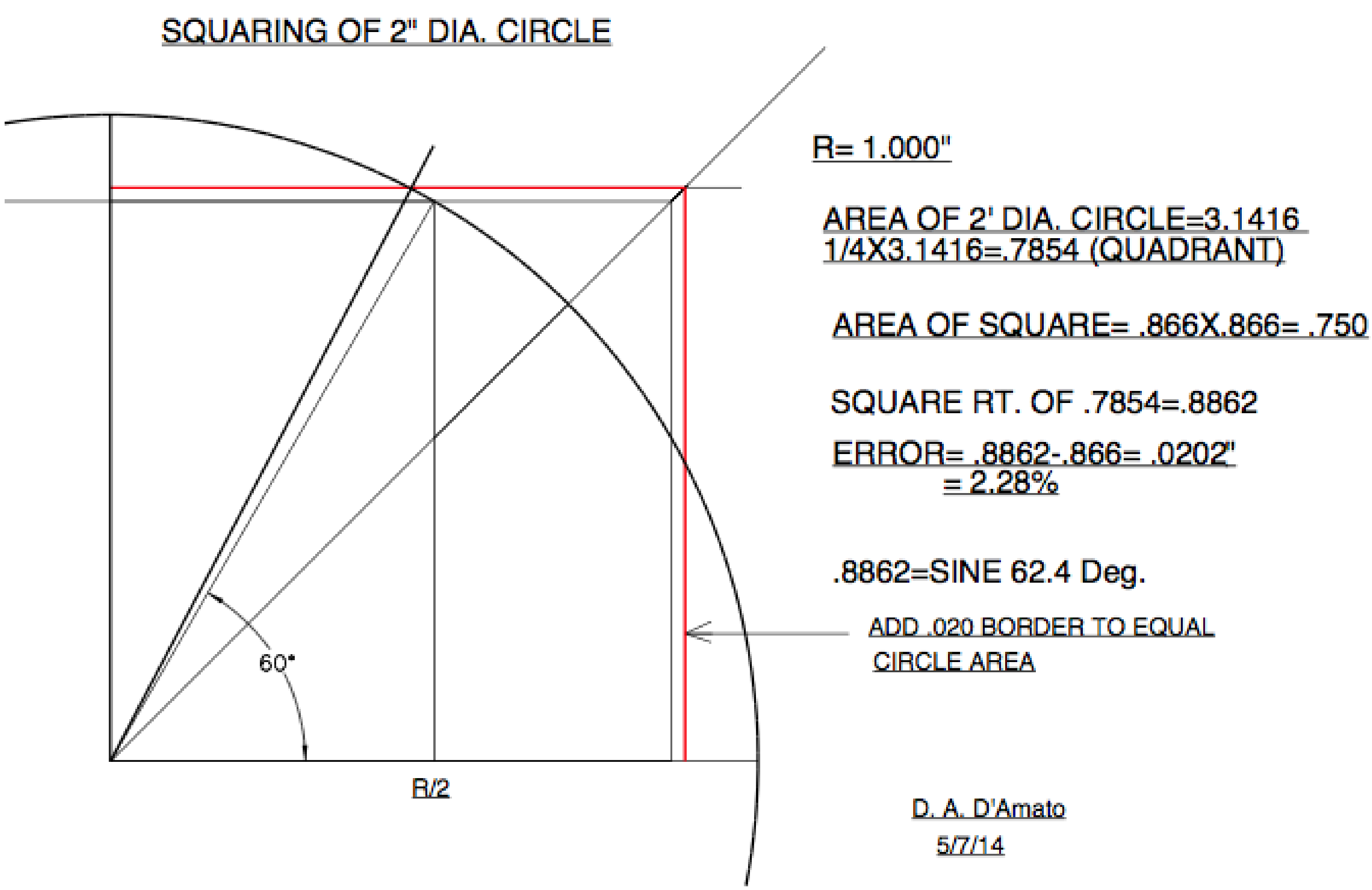
The expression "squaring the circle" is sometimes used as a metaphor for trying to do the impossible.[1]

Squaring The Circle: The excerpt from Wikipedia discusses the impossibility of squaring the circle and shows a drawing of what one should look like if, drawn with only an unmarked straight edge and compass. Having solved the problem of Trisecting an Angle, I decided to use a similar approach to this ancient puzzle and use transcendental functions (trigonometry) to solve the problem. The argument used is that Pi is transcendental and not an algebraic irrational number (funny, I always thought it to be a very long constant). In the example shown, except for setting the radius r=1, Pi is the only other number. The task is to draw the figure shown using the simple tools.

After looking at the graph, I noted the intersecting points on the circle and wondered what would I find if I drew a radial line to the intersecting point, in the first quadrant? As visually, that point appeared to be at 60 deg., the diagram was redrawn as shown in Fig. 17. It is simply using the known 60 deg. as the important radial angle to form an intersection of three lines. In this approach Pi is not the dominant factor but what is important is the point of intersection of the as yet unknown angle of the radial line with any circle or square!

As the calculations on the drawing show, a comparison of areas indicate that a 60 deg. square is smaller by 2.28%. By area calculation the inverse Sine of .8862 (value of ½ the square root of Pi) gives the critical angel to be 62.4 deg. But as we cannot measure any angle or distance with the simple tools, we can 5-sect angles. Fig. 18 shows that by 5-secting 15 deg., then the resulting 12 deg. angle, we get a segment of 2.4 deg., which we then add to the known 60 deg. angle to get the all-important **Constant 62.4** deg.

Fig. 17



Squaring the circle

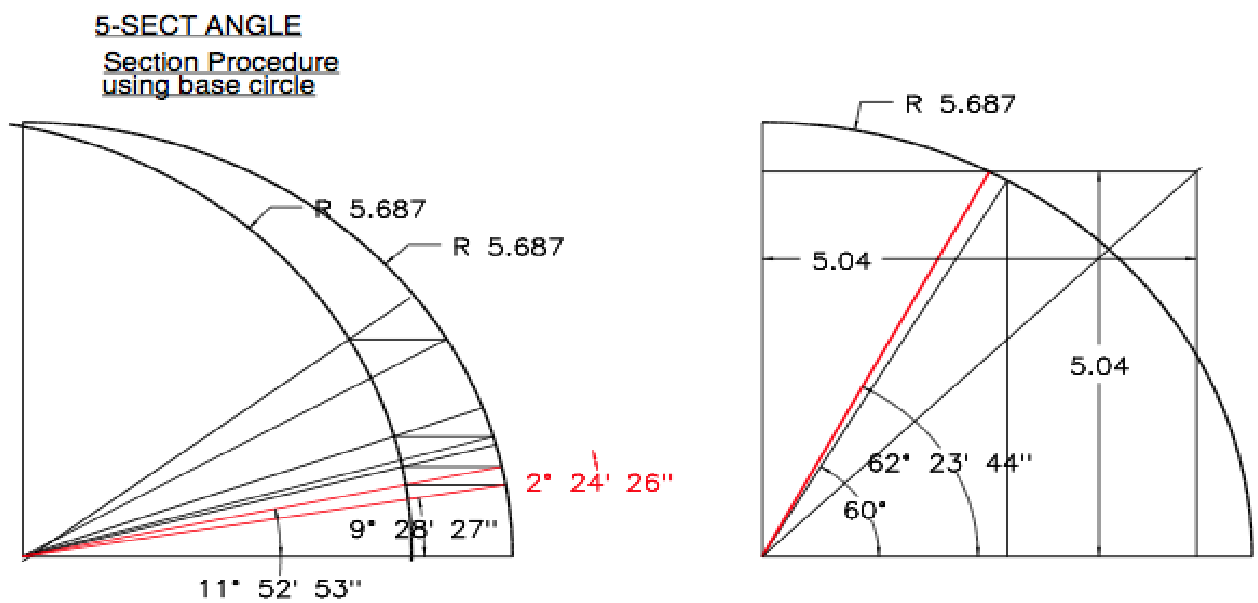


Fig. 18

The following drawings illustrate how an angle constant is used to square the circle and to circle the square!

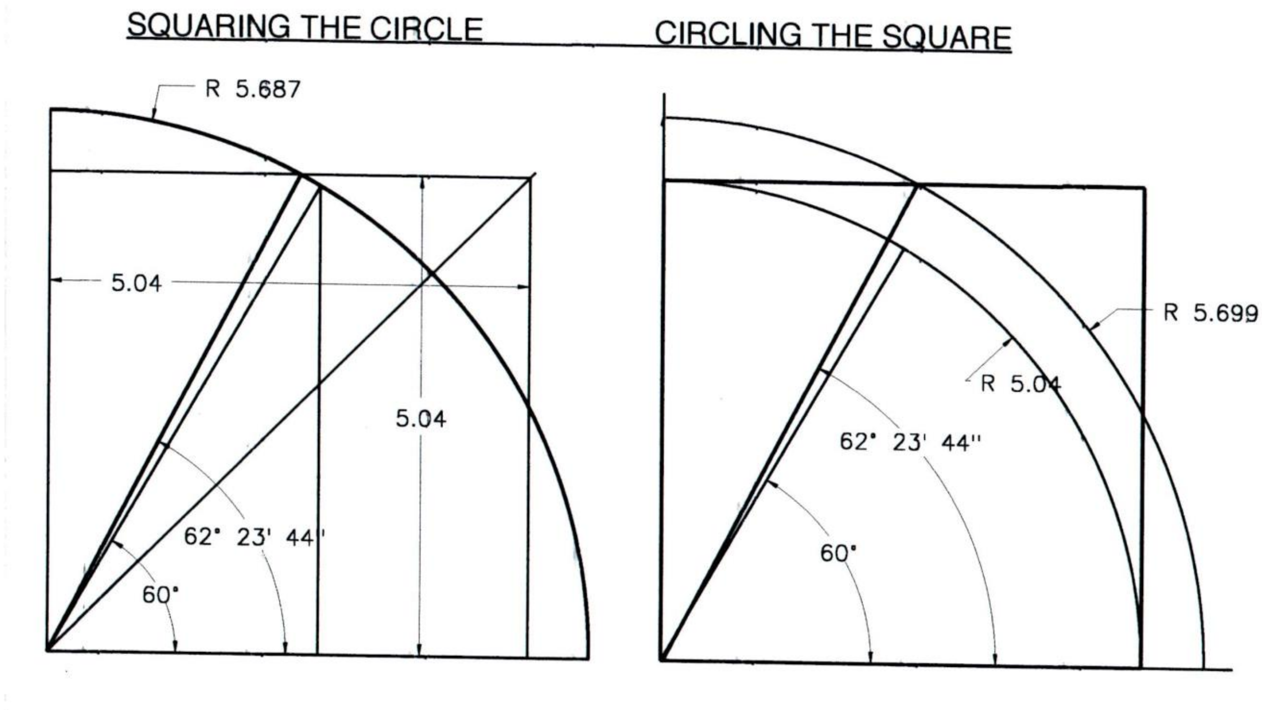
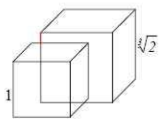


Fig. 19

Doubling the cube

From Wikipedia, the free encyclopedia

Jump to: navigation, search



A unit cube (side=1, volume=1) and a cube with twice the volume (side = $\sqrt[3]{2}$ = 1.2599210498948732..., volume=2).

Doubling the cube, also known as the Delian problem, is an ancient [1] geometric problem. Given the edge of a cube, the problem requires the construction of the edge of a second cube whose volume is double that of the first, using only the tools of a compass and straightedge. As with the related problems of squaring the circle and trisecting the angle, doubling the cube is now known to be impossible.

The Egyptians, Indians, and particularly the Greeks [2] were aware of the problem and made many futile attempts at solving what they saw as an obstinate but soluble problem. [3][4] However, the nonexistence of a solution was finally proven by Pierre Wantzel in 1837, applying the contemporary development of abstract algebra by Galois.

In algebraic terms, doubling a unit cube requires the construction of a line segment of length x, where $x^3 = 2$; in other words, $x = \sqrt[3]{2}$. This is because a cube of side length 1 has a volume of $1^3 = 1$, and a cube of twice that volume (a volume of 2) has a side length of the cube root of 2. The impossibility of doubling the cube is therefore equivalent to the statement that $\sqrt[3]{2}$ is not a constructible number. This is a consequence of the fact that the coordinates of a new point constructed by a compass and straightedge are roots of polynomials over the field generated by the coordinates of previous points, of no greater degree than a quadratic. This implies that the degree of the field generated by a constructible point must be a power of 2. The field generated by $\sqrt[3]{2}$, however, is of degree 3.

Doubling The Cube: Again, another unsolved ancient puzzle from a Wikipedia site (using only the simple tools). As in the previous problem, I have included excerpts from their paper in which this problem is described, which then proceeds to explain why it can't be done. None-the-less, I have found a solution to this 2,500-year-old puzzle and yet again, as in squaring the circle, the final step involves angle sectioning.

Solution procedure:

1. The first step is to draw the first quadrant of a circle equivalent in size to the 5-sected diagram used to square the circle.
2. From the apex draw a 45 deg. line to intersect and extend beyond the circular arc. Fig. 1.
3. From this point draw horizontal and vertical lines to form a square; where the sides, area and cube-volume, have a value of 1.
4. At this point we must construct another square whose sides must be increased to the value of the cube root of 2 or its dimensional value 1.2599210.
5. As was done in squaring the circle, after we calculate the magic angle to be 63deg. we now must find its point of intersection at the base circle.
6. Draw the known angle of 60 deg. to the base circle then 5-sect a 15 deg. angle to get the 3 deg. required. But this arc section has already been determined in the previous solution so we simply add this to the circle as shown.
7. From this point a horizontal line is drawn to the 45 deg. line and the square is completed for the solution.

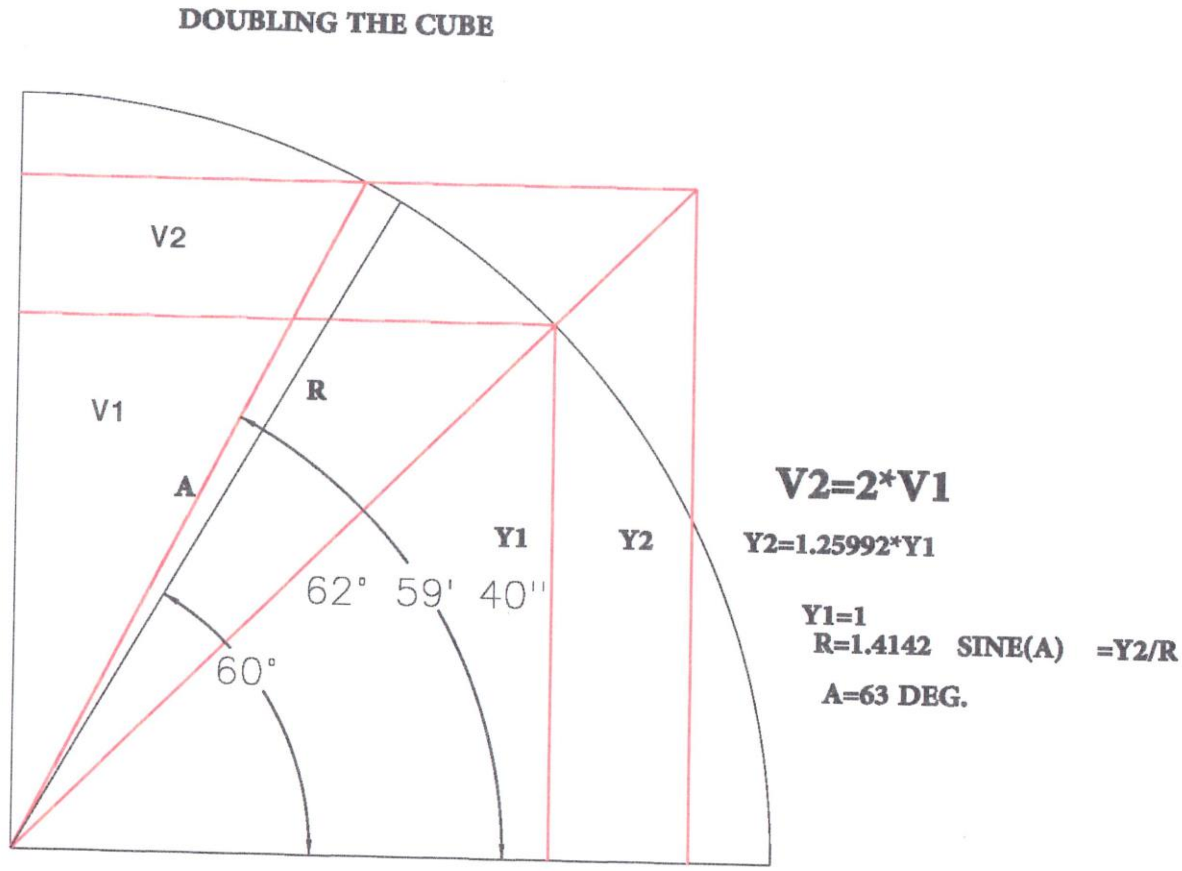


Fig. 20

It is interesting to note that the magic angle required for squaring the circle and for doubling the cube are less than 1 deg. apart and but for the ability to trisect and 5-sect any known angle, these problems could not have been solved with a high degree of accuracy. Further, an area or volume analysis using 60 deg. as the starting point a trial and error solution to reach the proper angle is evidently possible. This approach would eliminate the need for the factors 2 and Pi.

The Curious Case of Pi: Having been determined to be the ratio of the circumference of a circle over the diameter of that circle - and an infinitely long irrational number - Pi to mathematicians - has been the subject of study for hundreds of years. Apparently the purpose of the studies has been to determine if this can be evaluated algebraically or by the ratio of whole numbers e.g. 22/7- which is only accurate to two decimal places. Wikipedia presents studies of Pi. In the paper on squaring the circle, where this puzzle is promptly dismissed as being insolvable the dissertation that follows is primarily about Pi.

Apparently Pi is an important constant and has been shown to be transcendental - although there are those who are still trying to find algebraic equations to evaluate Pi. If you find this confusing so do I, whenever I had to use it, it was simply a constant nothing more. Still, however senseless these attempts to find ways to calculate Pi seem to be, I again bowed to my curiosity. Having constructed Pi in solving the squaring the circle I decided to calculate Pi using two separate methods - algebraic and transcendental. Both simple equations shown below give results with matching Pi decimal points in greater number than those shown in Wikipedia.

Note that the results of the transcendental equation match the value of Pi to the calculator's capacity. However, eleven decimal points is significant.

Equation 1: Transcendental

$$360000 \times 180 \times \tan(1 \div 360000)$$

$$360000 \times 180 \times (\tan(1 \div 360000)) =$$

$$3.14159265359$$

Equation 2: Algebraic

$$3 + .1 \times \sqrt{(2.004848)}$$

$$3 + .1 \times \sqrt{(2.004848)} =$$

$$3.14159265518$$

Pi:

$$\pi =$$

$$3.14159265359$$

Summary of Results and Conclusions:

In a number of technical papers I have written and presented, this section of the papers were easily written as the subject matters were directed to receptive audiences looking for solutions to their machine or manufacturing problems. In this paper I present solutions to 2,500 years old drafting problems universally believed to be unsolvable. These are; Trisecting an Angle, Squaring the Circle, Doubling the Cube and Constructing a Nine Sided Polygon. They are puzzles because the solutions must be constructed using only a compass and an unmarked straight edge. They are problems because no one has ever solved them.

So what is the problem if - as the paper shows - you have the solutions? It is that the thesis is instructive as well by presenting methodological procedures and analytical approaches to these puzzles. Thus this aspect of the paper becomes as important as the solutions. Then how does one summarize an analytical approach to a problem where the solution subject is dominant?

Add to this, that up to now these puzzles were treated individually, here we have a unique situation in that if you cannot trisect an angle then you cannot solve the other “unsolvable” problems. Thus it was pure chance that I stumbled on the problem of trisecting an angle first.

The trisection solution became evident after a brief study of a trisected angle constructed in the first quadrant of a base circle. By simply drawing a single horizontal line between the third line and the fourth line of that triangle it was obvious that this intersection point was singular to that trisected angle. By developing a number of such points one then simply connects the points to produce a Trisection curve. Moreover, similar curves can be drawn for any division of an angle. While the trisection curve is drawn only in the first quadrant, it can be used to section angles greater than 90 deg. And for those mathematicians who tried and failed to solve this problem the plotted results of derived equations are shown.

Much is presented on the use of a base circle and the inherent properties of the first quadrant of that circle. Both are required as well as other sectioning curves to solve all of the other “unsolvable” puzzles. All require a base circle and known angles drawn in the first quadrant.

In the solution to squaring the circle - again - the intersection of lines is the clue to the problem. Here the puzzle was presented as a diagram with a circle and a superimposed square both having an area equal to Pi. However, with the tools allowed, one cannot measure or draw the square root of Pi, therefore one cannot solve the puzzle. The solution presented here was to simply draw an angular line from the apex of the first quadrant to the intersection of the circle and square- basically eliminating Pi as a factor. Simple calculations show this critical angle to be 62.4 deg. and the point of intersection on the circle was located by using the 5-secting of angles process. Basically, once this angel is drawn any point on this angle is an intersection point for squaring the circle.

Doubling the cube used a similar approach by adding a base circle to the construction in order to determine the critical angle of 63 deg. - essentially the solution to the cube problem.

Drawing the nine-sided polygon was accomplished by trisecting a 60 deg. angle, to get the 40 deg. required for the solution.

From Wikipedia and other sources, it appears that for anyone trying to solve the trisection problem, the target accuracy was set at 0.1 deg. at any angle. This is not an acceptable error or solution. Only the perfect logical solution submitted here has a zero-degree error for any angle from 0 to 360 deg. However, with the other problems, where we are dealing with transcendental equations and irrational numbers, how exact must the answers be and how many decimal places of Pi should or can be used for the drafting puzzle Squaring the Circle? Here we show exact area results, using four decimal places.

Finally, methods of calculating Pi are submitted - apparently another subject of fanaticism noted by Wikipedia.

Dominic A. D'Amato
June 27, 2016